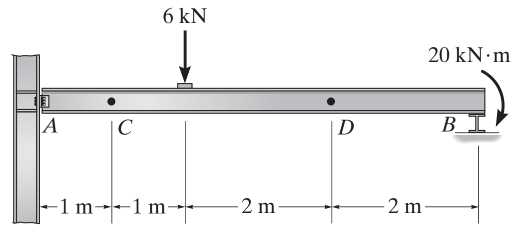
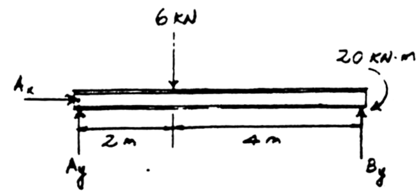
4–1. Determine the internal normal force, shear force, and bending moment in the beam at points C and D. Assume the support at A is a pin and B is a roller.



Entire beam:



$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x = 0$

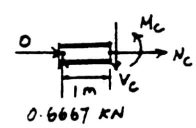
$\circlearrowleft + \sum M_A = 0;$ $\quad B_y(6) - 20 - 6(2) = 0$

$B_y = 5.333 \text{ kN}$

$+\uparrow \sum F_y = 0;$ $\quad A_y + 5.333 - 6 = 0$

$A_y = 0.6667 \text{ kN}$

Segment AC:



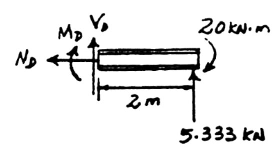
$\xrightarrow{+} \sum F_x = 0;$ $\quad N_C = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad 0.6667 - V_C = 0$

$V_C = 0.667 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_C = 0;$ $\quad M_C - 0.6667(1) = 0$

$M_C = 0.667 \text{ kN} \cdot \text{m}$ **Ans.**



Segment DB:

$\xrightarrow{+} \sum F_x = 0;$ $\quad N_D = 0$ **Ans.**

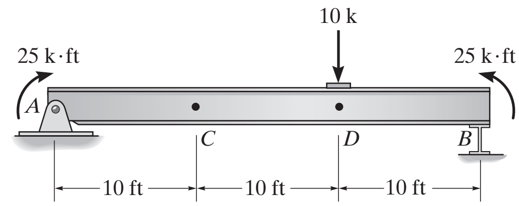
$+\uparrow \sum F_y = 0;$ $\quad V_D + 5.333 = 0$

$V_D = -5.33 \text{ kN}$ **Ans.**

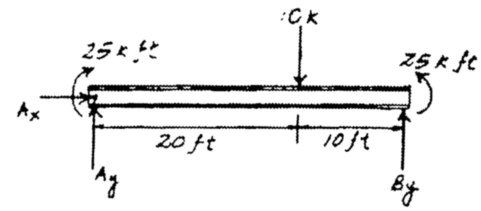
$\circlearrowleft + \sum M_D = 0;$ $\quad -M_D + 5.333(2) - 20 = 0$

$M_D = -9.33 \text{ kN} \cdot \text{m}$ **Ans.**

4–2. Determine the internal normal force, shear force, and bending moment in the beam at points C and D. Assume the support at B is a roller. Point D is located just to the right of the 10–k load.



Entire Beam:



$\circlearrowleft + \sum M_A = 0;$ $B_y(30) + 25 - 25 - 10(20) = 0$

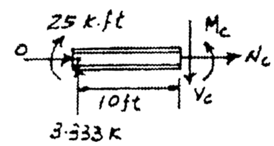
$B_y = 6.667$ k

$+\uparrow \sum F_y = 0;$ $A_y + 6.667 - 10 = 0$

$A_y = 3.333$ k

$\overset{+}{\rightarrow} \sum F_x = 0;$ $A_x = 0$

Segment AC:



$\overset{+}{\rightarrow} \sum F_x = 0;$ $N_C = 0$ **Ans.**

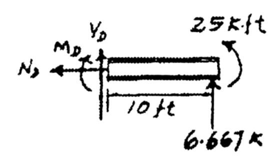
$+\uparrow \sum F_y = 0;$ $-V_C + 3.333 = 0$

$V_C = 3.33$ k **Ans.**

$\circlearrowleft + \sum M_C = 0;$ $M_C - 25 - 3.333(10) = 0$

$M_C = 58.3\ \text{k} \cdot \text{ft}$ **Ans.**

Segment DB:



$\overset{+}{\rightarrow} \sum F_x = 0;$ $N_D = 0$ **Ans.**

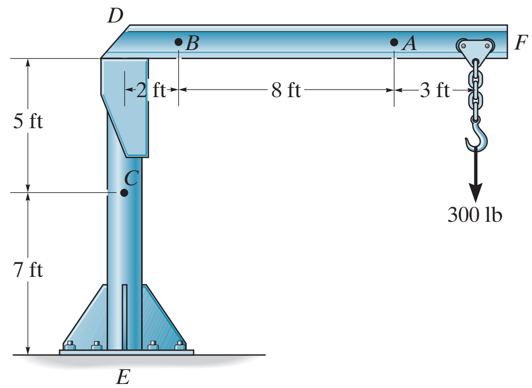
$+\uparrow \sum F_y = 0;$ $V_D + 6.6667 = 0$

$V_D = -6.67$ k **Ans.**

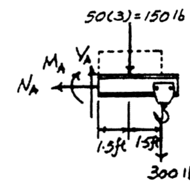
$\circlearrowleft + \sum M_D = 0;$ $-M_D + 25 + 6.667(10) = 0$

$M_D = 91.7\ \text{k} \cdot \text{ft}$ **Ans.**

4–3. The boom *DF* of the jib crane and the column *DE* have a uniform weight of 50 lb/ft. If the hoist and load weigh 300 lb, determine the internal normal force, shear force, and bending moment in the crane at points *A*, *B*, and *C*.



Equations of Equilibrium: For point *A*



$\overset{+}{\leftarrow} \sum F_x = 0;$ $\quad N_A = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad V_A - 150 - 300 = 0$

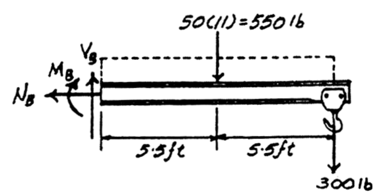
$V_A = 450 \text{ lb}$ **Ans.**

$\circlearrowleft + \sum M_A = 0;$ $\quad -M_A - 150(1.5) - 300(3) = 0$

$M_A = -1125 \text{ lb} \cdot \text{ft} = -1.125 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicates that M_A acts in the opposite direction to that shown on FBD.

Equations of Equilibrium: For point *B*



$\overset{+}{\leftarrow} \sum F_x = 0;$ $\quad N_B = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad V_B - 550 - 300 = 0$

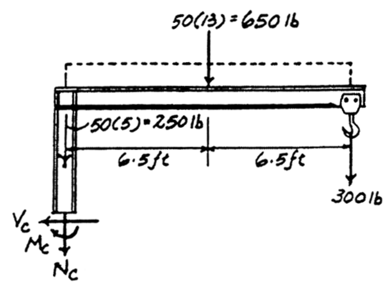
$V_B = 850 \text{ lb}$ **Ans.**

$\circlearrowleft + \sum M_B = 0;$ $\quad -M_B - 550(5.5) - 300(11) = 0$

$M_B = -6325 \text{ lb} \cdot \text{ft} = -6.325 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicates that M_B acts in the opposite direction to that shown on FBD.

Equations of Equilibrium: For point *C*



$\overset{+}{\leftarrow} \sum F_x = 0;$ $\quad V_C = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad -N_C - 250 - 650 - 300 = 0$

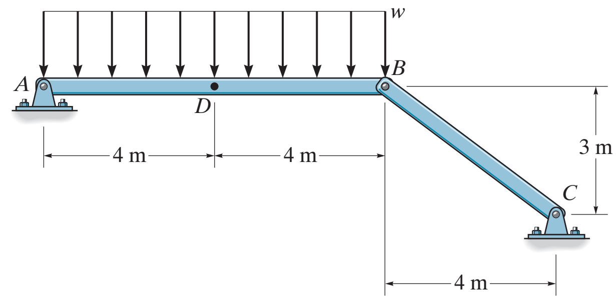
$N_C = -1200 \text{ lb} = -1.20 \text{ kip}$ **Ans.**

$\circlearrowleft + \sum M_C = 0;$ $\quad -M_C - 650(6.5) - 300(13) = 0$

$M_C = -8125 \text{ lb} \cdot \text{ft} = -8.125 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicate that N_C and M_C act in the opposite direction to that shown on FBD.

***4–4.** Determine the internal normal force, shear force, and bending moment at point D. Take $w = 150$ N/m.



$\circlearrowleft + \sum M_A = 0;$ $\quad -150(8)(4) + \frac{3}{5} F_{BC}(8) = 0$

$F_{BC} = 1000$ N

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad A_x - \frac{4}{5}(1000) = 0$

$A_x = 800$ N

$+\uparrow \sum F_y = 0;$ $\quad A_y - 150(8) + \frac{3}{5}(1000) = 0$

$A_y = 600$ N

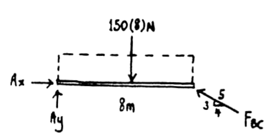
$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad N_D = -800$ N **Ans.**

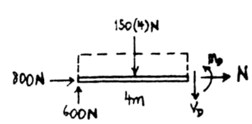
$+\uparrow \sum F_y = 0;$ $\quad 600 - 150(4) - V_D = 0$

$V_D = 0$ **Ans.**

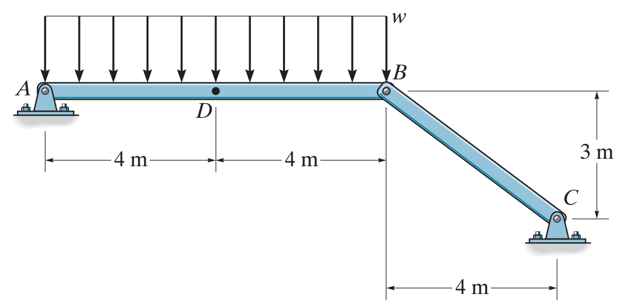
$\circlearrowleft + \sum M_D = 0;$ $\quad -600(4) + 150(4)(2) + M_D = 0$

$M_D = 1200 \text{ N} \cdot \text{m} = 1.20 \text{ kN} \cdot \text{m}$ **Ans.**





4–5. The beam AB will fail if the maximum internal moment at D reaches 800 N·m or the normal force in member BC becomes 1500 N. Determine the largest load w it can support.



Assume maximum moment occurs at D;

$\zeta + \sum M_D = 0;$ $\quad M_D - \frac{8w}{2}(4) + 4w\,(2) = 0$

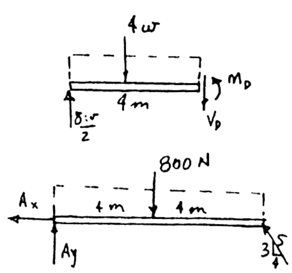
$800 = 8\,w$

$w = 100 \text{ N/m}$

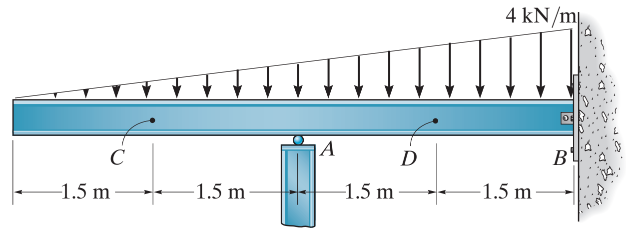
$\zeta + \sum M_A = 0;$ $\quad -800(4) + T_{BC}(0.6)(8) = 0$

$T_{BC} = 666.7 \text{ N} < 1500 \text{ N}$ **(O. K!)**

$w = 100 \text{ N/m}$ **Ans.**



4–6. Determine the internal normal force, shear force, and bending moment in the beam at points C and D. Assume the support at A is a roller and B is a pin.



Support Reactions. Referring to the FBD of the entire beam in Fig. *a*,

$$\circlearrowleft + \sum M_B = 0; \quad \frac{1}{2}(4)(6)(2) - A_y(3) = 0 \quad A_y = 8 \text{ kN}$$

Internal Loadings. Referring to the FBD of the left segment of the beam sectioned through point C, Fig. *b*,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad N_C = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \sum F_y = 0; \quad -\frac{1}{2}(1)(1.5) - V_C = 0 \quad V_C = -0.75 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_C = 0; \quad M_C + \frac{1}{2}(1)(1.5)(0.5) = 0 \quad M_C = -0.375 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

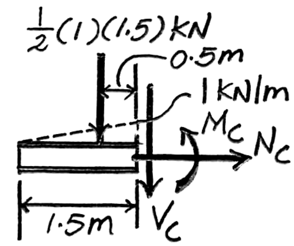
Referring to the FBD of the left segment of the beam sectioned through point D, Fig. *c*,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad N_D = 0 \qquad \textbf{Ans.}$$

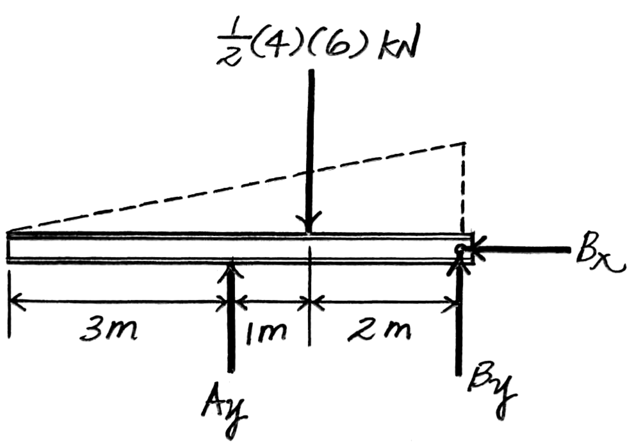
$$+\uparrow \sum F_y = 0; \quad 8 - \frac{1}{2}(3)(4.5) - V_D = 0 \quad V_D = 1.25 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_D = 0; \quad M_D + \frac{1}{2}(3)(-4.5)(1.5) - 8(1.5) = 0$$

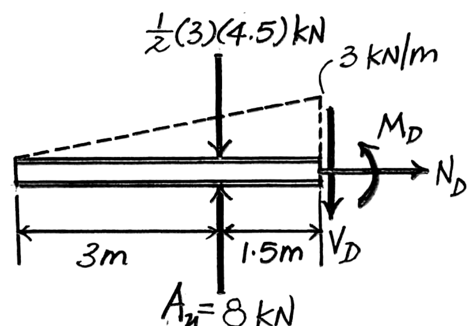
$$M_D = 1.875 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(b)

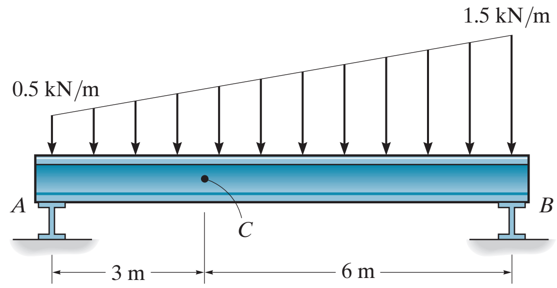


(a)



(c)

4–7. Determine the internal normal force, shear force, and bending moment at point C. Assume the reactions at the supports A and B are vertical.



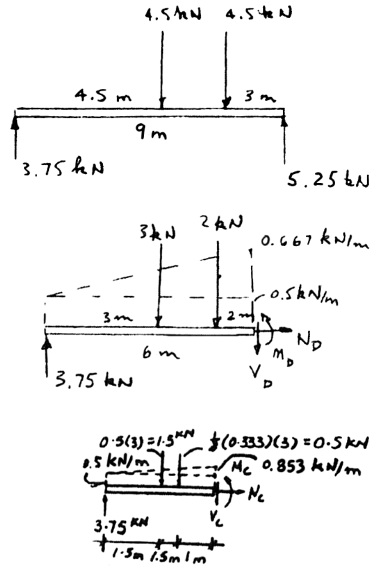
$\xrightarrow{+} \sum F_x = 0;$ $N_C = 0$ **Ans.**

$+\downarrow \sum F_y = 0;$ $V_C + 0.5 + 1.5 - 3.75 = 0$

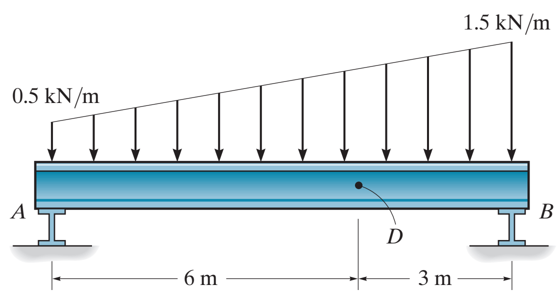
$V_C = 1.75 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_C = 0;$ $M_C + 0.5(1) + 1.5(1.5) - 3.75(3) = 0$

$M_C = 8.50 \text{ kN} \cdot \text{m}$ **Ans.**



***4–8.** Determine the internal normal force, shear force, and bending moment at point D. Assume the reactions at the supports A and B are vertical.



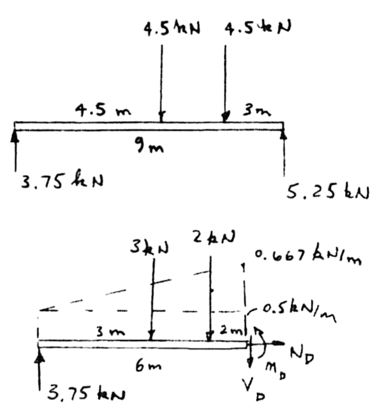
$\xrightarrow{+} \sum F_x = 0;$ $N_D = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $3.75 - 3 - 2 - V_D = 0$

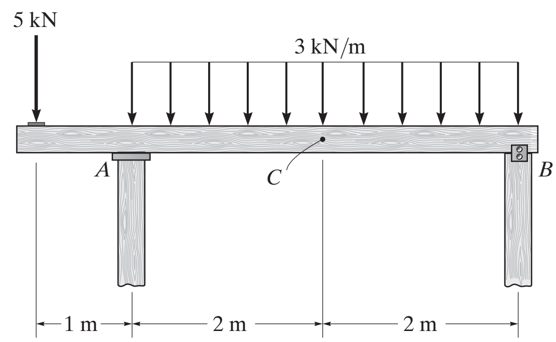
$V_D = -1.25 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_D = 0;$ $M_D + 2(2) + 3(3) - 3.75(6) = 0$

$M_D = 9.50 \text{ kN} \cdot \text{m}$ **Ans.**



4–9. Determine the internal normal force, shear force, and bending moment in the beam at point *C*. The support at *A* is a roller and *B* is pinned.



Support Reactions. Referring to the FBD of the entire beam in Fig *a*,

$\circlearrowleft + \sum M_A = 0; \quad B_y(4) + 5(1) - 3(4)(2) = 0 \quad B_y = 4.75 \text{ kN}$

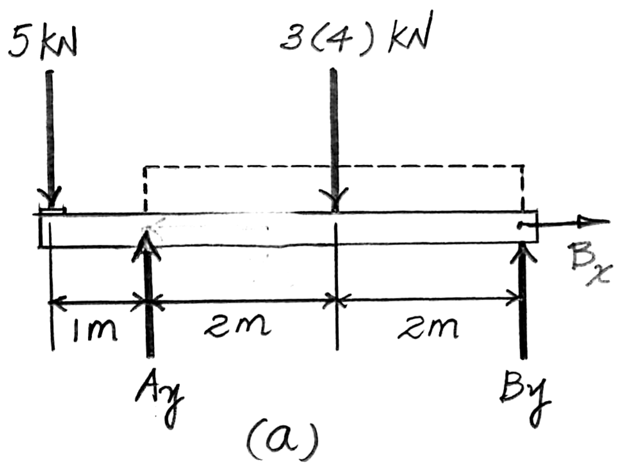
$\overset{+}{\rightarrow} \sum F_x = 0; \quad B_x = 0$

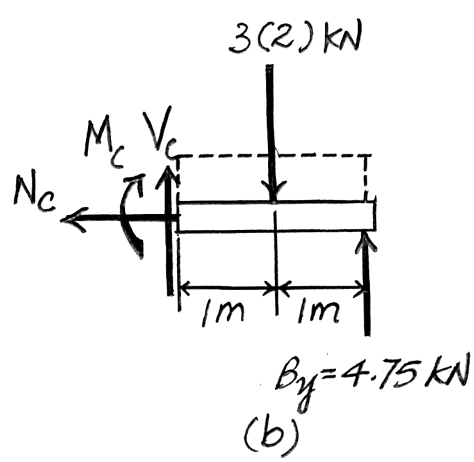
Internal Loadings. Referring to the FBD of the right segment of the beam sectioned through point *c*, Fig. *b*,

$\overset{+}{\rightarrow} \sum F_x = 0; \quad N_C = 0$ **Ans.**

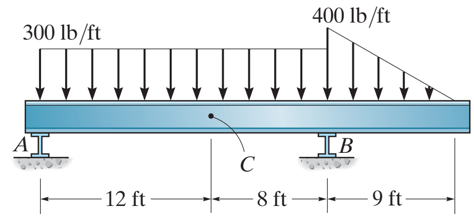
$+\uparrow \sum F_y = 0; \quad V_C + 4.75 - 3(2) = 0 \quad V_C = 1.25 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_C = 0; \quad 4.75(2) - 3(2)(1) - M_C = 0 \quad M_C = 3.50 \text{ kN} \cdot \text{m}$ **Ans.**





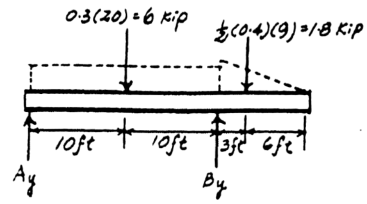
4–10. Determine the internal normal force, shear force, and bending moment at point C. Assume the reactions at the supports A and B are vertical.



Support Reactions:

$\zeta + \sum M_A = 0;$ $\quad B_y(20) - 6(10) - 1.8(23) = 0$

$B_y = 5.07 \text{ kip}$



$+\uparrow \sum F_y = 0;$ $\quad A_y + 5.07 - 6 - 1.8 = 0$

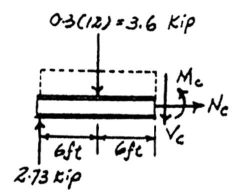
$A_y = 2.73 \text{ kip}$

Equations of Equilibrium: For point C

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad N_C = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad 273 - 3.60 - V_C = 0$

$V_C = -0.870 \text{ kip}$ **Ans.**

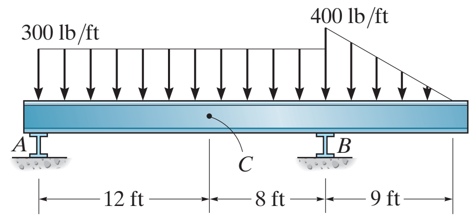


$\zeta + \sum M_C = 0;$ $\quad M_C + 3.60(6) - 2.73(12) = 0$

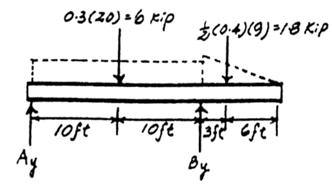
$M_C = 11.2 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicates that V_C acts in the opposite direction to that shown on FBD.

4–11. Determine the internal normal force, shear force, and bending moment at points D and E. Assume the reactions at the supports A and B are vertical.



Support Reactions:



$\zeta + \sum M_A = 0;$ $B_y(20) - 6(10) - 1.8(23) = 0$

$B_y = 5.07$ kip

$+\uparrow \sum F_y = 0;$ $A_y + 5.07 - 6 - 1.8 = 0$

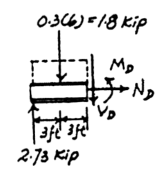
$A_y = 2.73$ kip

Equations of Equilibrium: For point D

$\overset{+}{\rightarrow} \sum F_x = 0;$ $N_D = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $2.73 - 1.8 - V_D = 0$

$V_D = 0.930$ kip **Ans.**



$\zeta + \sum M_D = 0;$ $M_D + 1.8(3) - 2.73(6) = 0$

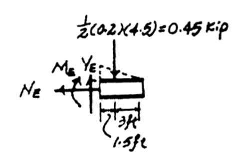
$M_D = 11.0 \text{ kip} \cdot \text{ft}$ **Ans.**

Equations of Equilibrium: For point E

$\overset{+}{\leftarrow} \sum F_x = 0;$ $N_E = 0$ **Ans.**

$+\uparrow \sum F_y = 0;$ $V_E - 0.45 = 0$

$V_E = 0.450$ kip **Ans.**

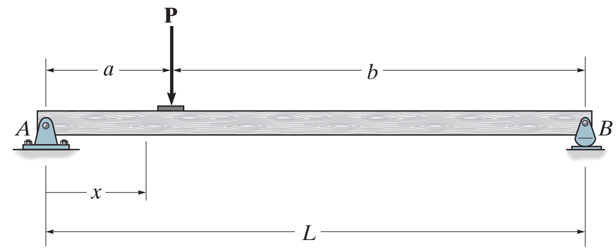


$\zeta + \sum M_E = 0;$ $-M_E - 0.45(1.5) = 0$

$M_E = -0.675 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicates that M_E acts in the opposite direction to that shown on FBD.

***4–12.** Determine the shear and moment throughout the beam as a function of x.



Support Reactions: Referring to the FBD of the entire beam in Fig. a,

$\zeta + \sum M_A = 0;\qquad N_B(L) - Pa = 0 \qquad N_B = \dfrac{Pa}{L}$

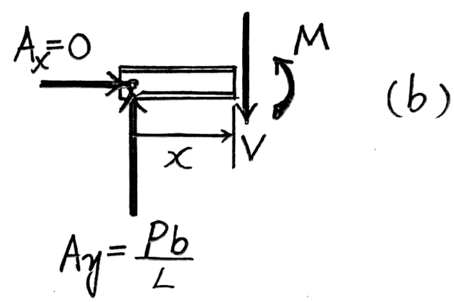
$\zeta + \sum M_B = 0;\qquad Pb - A_y(L) = 0 \qquad A_y = \dfrac{Pb}{L}$

$\overset{+}{\rightarrow} \sum F_x = 0;\qquad A_x = 0$

Internal Loading: For $0 \le x < a$, refer to the FBD of the left segment of the beam in Fig. b.

$+\uparrow \sum F_y = 0;\qquad \dfrac{Pb}{L} - V = 0 \qquad V = \dfrac{Pb}{L}$ **Ans.**

$\zeta + \sum M_O = 0;\qquad M - \dfrac{Pb}{L}x = 0 \quad M = \dfrac{Pb}{L}x$ **Ans.**

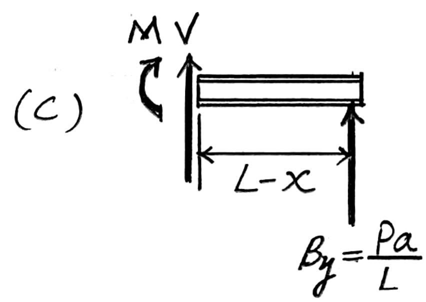


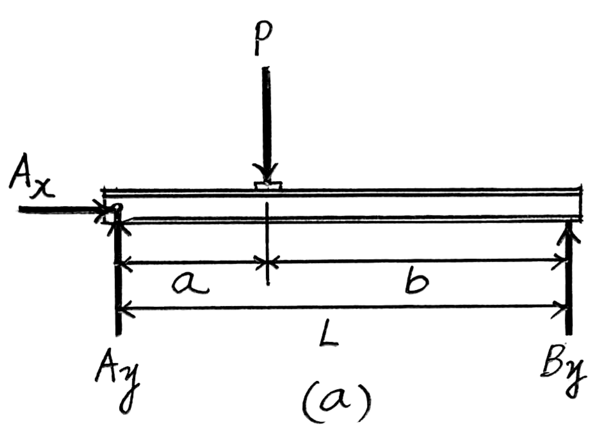
For $a < x \le L$, refer to the FBD of the right segment of the beam in Fig. c.

$+\uparrow \sum F_y = 0;\qquad V + \dfrac{Pa}{L} = 0 \qquad V = -\dfrac{Pa}{L}$ **Ans.**

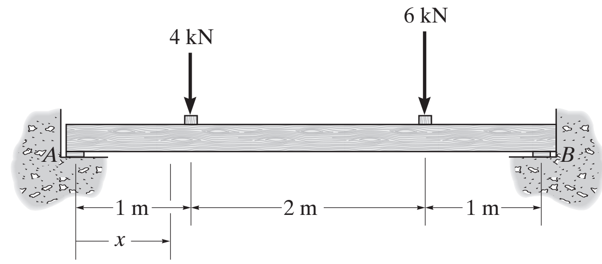
$\zeta + \sum M_O = 0;\qquad \dfrac{Pa}{L}(L - x) - M = 0$

$$M = \frac{Pa}{L}(L - x)$$ **Ans.**





4–13. Determine the shear and moment in the floor girder as a function of x. Assume the support at A is a pin and B is a roller.



Support Reactions: Referring to the FBD of the entire beam in Fig. *a*.

$\circlearrowleft + \sum M_A = 0; \quad B_y(4) - 4(1) - 6(3) = 0$

$B_y = 5.50 \text{ kN}$

$\circlearrowleft + \sum M_B = 0; \quad 6(1) + 4(3) - A_y(4) = 0$

$A_y = 4.50 \text{ kN}$

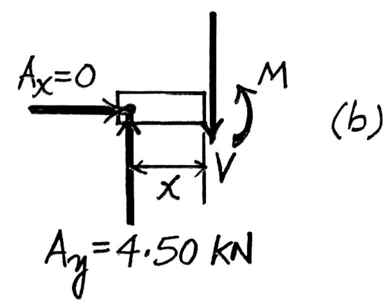
$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x = 0$

Internal Loadings: For $0 \le x < 1$ m, Referring to the FBD of the left segment of the beam in Fig. *b*,

$+\uparrow \sum F_y = 0; \quad 4.50 - V = 0 \qquad V = 4.50 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_O = 0; \quad M - 4.50x = 0$

$M = \{4.50x\} \text{ kN} \cdot \text{m}$ **Ans.**



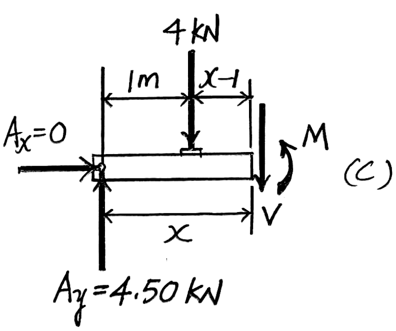
For $1 \text{ m} < x < 3$ m, referring to the FBD of the left segment of the beam in Fig. *c*,

$+\uparrow \sum F_y = 0; \quad 4.50 - 4 - V = 0$

$V = 0.500 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_O = 0; \quad M + 4(x - 1) - 4.50x = 0$

$M = \{0.5x + 4\} \text{ kN} \cdot \text{m}$ **Ans.**

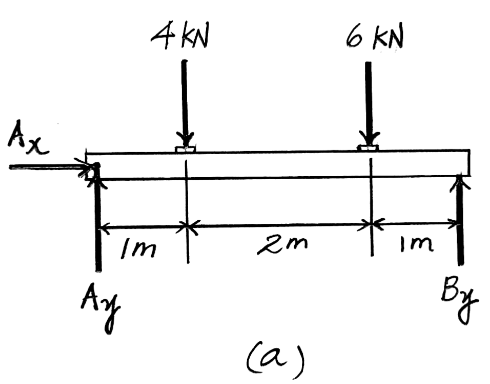


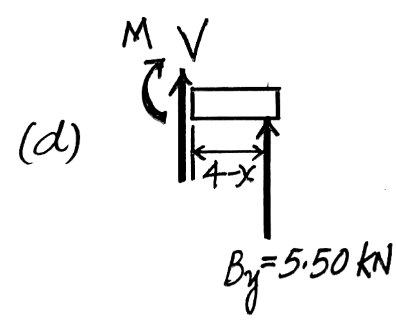
For $3 \text{ m} < x \le 4$ m, referring to the FBD of the right segment of the beam in Fig. *d*,

$+\uparrow \sum F_y = 0; \quad V + 5.50 = 0 \qquad V = -5.50 \text{ kN}$ **Ans.**

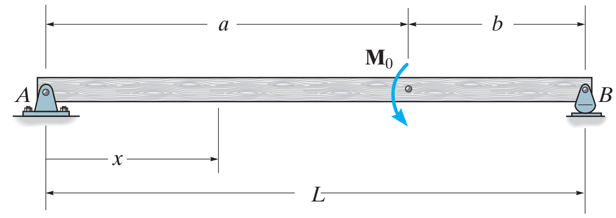
$\circlearrowleft + \sum M_O = 0; \quad 5.50(4 - x) - M = 0$

$M = \{-5.50x + 22\} \text{ kN} \cdot \text{m}$ **Ans.**





4–14. Determine the shear and moment throughout the beam as a function of x.



Support Reactions: Referring to the FBD of the entire beam in Fig. *a*

$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad A_x = 0$$

$$\circlearrowleft + \sum M_A = 0; \qquad M_O - N_B(L) = 0 \qquad B_y = \frac{M_O}{L}$$

$$\circlearrowleft + \sum M_B = 0; \qquad M_O - A_y(L) = 0 \qquad A_y = \frac{M_O}{L}$$

Internal Loadings: For $0 \le x < a$, refer to the FBD of the left segment of the beam is Fig. *b*.

$$+\uparrow \sum F_y = 0; \qquad \frac{M_O}{L} - V = 0 \qquad V = \frac{M_O}{L} \qquad \textbf{Ans.}$$

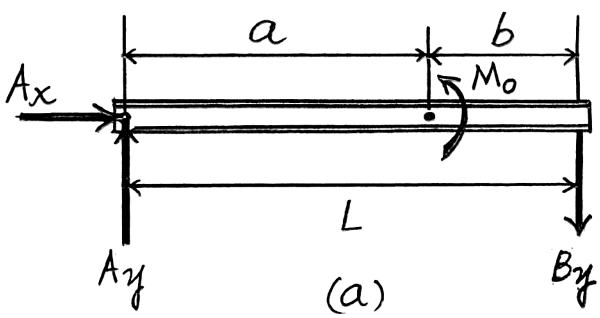
$$\circlearrowleft + \sum M_o = 0; \qquad M - \frac{M_o}{L}x = 0 \qquad M = \frac{M_O}{L}x \qquad \textbf{Ans.}$$

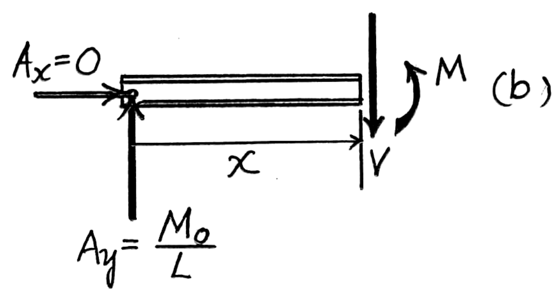
For $a < x \le L$, refer to the FBD of the right segment of the beam in Fig. *c*

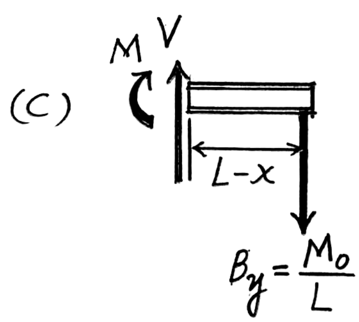
$$+\uparrow \sum F_y = 0; \qquad V - \frac{M_O}{L} = 0 \qquad V = \frac{M_O}{L} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_o = 0; \qquad -M - \frac{M_o}{L}(L - x) = 0 \qquad \textbf{Ans.}$$

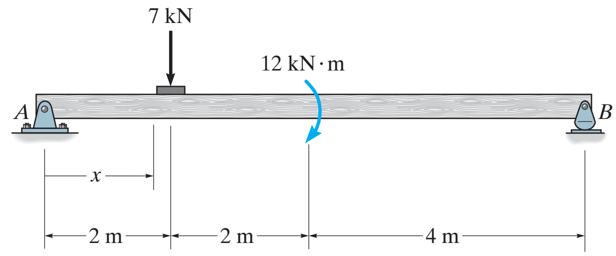
$$M = -\frac{M_o}{L}(L - x) \qquad \textbf{Ans.}$$



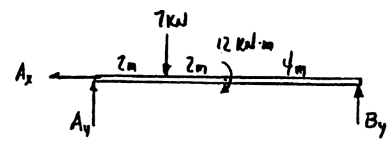




4–15. Determine the shear and moment throughout the beam as a function of x.



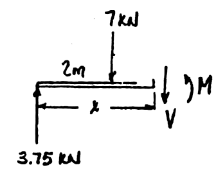
Reaction at A:



$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x = 0$ **Ans.**

$\circlearrowleft + \sum M_B = 0;$ $\quad A_y(8) - 7(6) + 12 = 0; \quad A_y = 3.75 \text{ kN}$

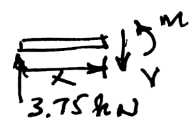
$0 \leq x < 2$ m



$+\uparrow \sum F_y = 0;$ $\quad 3.75 - V = 0;$ $\quad V = 3.75 \text{ kN}$ **Ans.**

$\circlearrowleft + \sum M = 0;$ $\quad 3.75x - M = 0;$ $\quad M = 3.75x \text{ kN}$ **Ans.**

$2 \text{ m} < x < 4 \text{ m}$

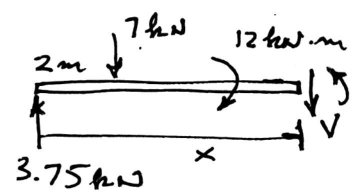


Segment:

$+\uparrow \sum F_y = 0;$ $\quad -V + 3.75 - 7 = 0;$ $\quad V = -3.25$ **Ans.**

$\circlearrowleft + \sum M = 0;$ $\quad -M + 3.75x - 7(x - 2) = 0;$

$M = -3.25x + 14$ **Ans.**



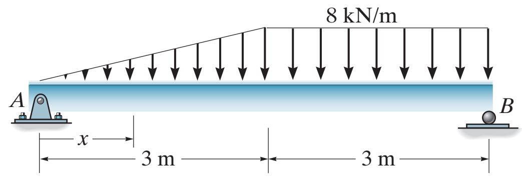
$4 \text{ m} < x \leq 8 \text{ m}$

$+\uparrow \sum F_y = 0;$ $\quad 3.75 - 7 - V = 0;$ $\quad V = -3.25 \text{ kN}$ **Ans.**

$\circlearrowright + \sum M = 0;$ $\quad -3.75x + 7(x - 2) - 12 + M = 0;$

$M = 26 - 3.25x$ **Ans.**

***4–16.** Determine the shear and moment throughout the beam as a function of x.



Support Reactions. Referring to the FBD of the entire beam in Fig. a,

$$\zeta + \sum M_A = 0; \quad B_y(6) - 8(3)(4.5) - \frac{1}{2}(8)(3)(2) = 0$$

$$B_y = 22 \text{ kN}$$

$$\zeta + \sum M_B = 0; \quad 8(3)(1.5) + \frac{1}{2}(8)(3)(4) - A_y(6) = 0$$

$$A_y = 14 \text{ kN}$$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x = 0$$

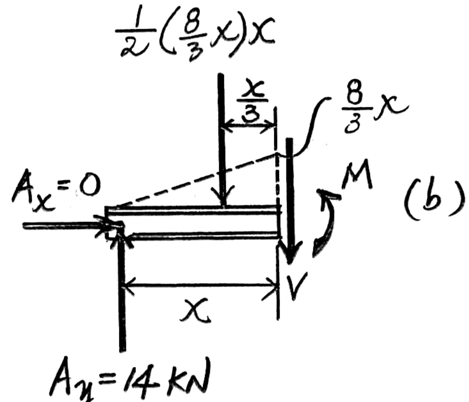
Internal Loadings: For $0 \le x < 3$ m, refer to the FBD of the left segment of the beam in Fig. b,

$$+\uparrow \sum F_y = 0; \quad 14 - \frac{1}{2}\left(\frac{8}{3}x\right)x - V = 0$$

$$V = \left\{-1.33x^2 + 14\right\} \text{ kN} \qquad \textbf{Ans.}$$

$$\zeta + \sum M_O = 0; \quad M + \frac{1}{2}\left(\frac{8}{3}x\right)(x)\left(\frac{x}{3}\right) - 14x = 0$$

$$M = \left\{-0.444\,x^3 + 14\,x\right\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



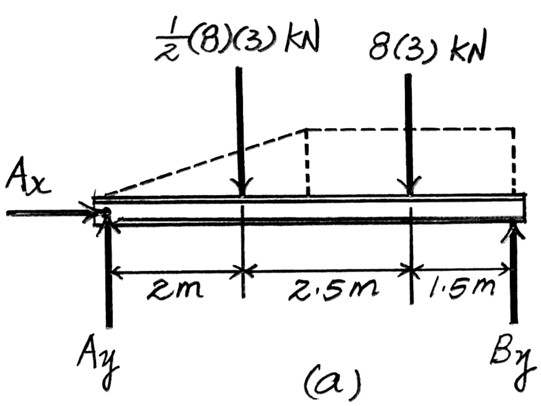
For 3 m $< x \le 6$ m, refer to the FBD of the right segment of the beam in Fig. c

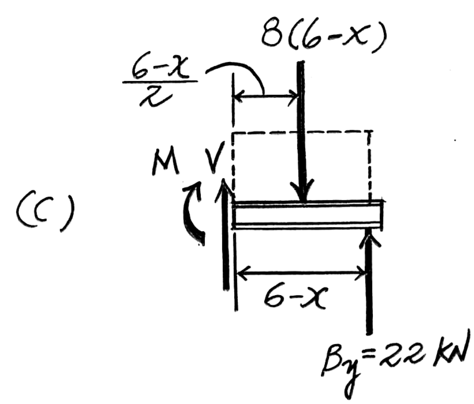
$$+\uparrow \sum F_y = 0; \quad V + 22 - 8(6 - x) = 0$$

$$V = \left\{-8x + 26\right\} \text{ kN} \qquad \textbf{Ans.}$$

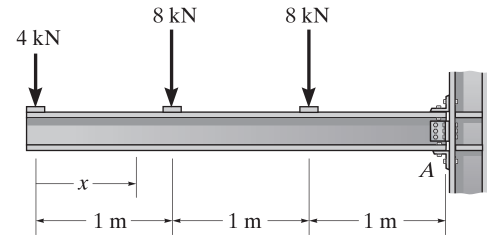
$$\zeta + \sum M_O = 0; \quad 22(6 - x) - 8(6 - x)\left(\frac{6 - x}{2}\right) - M = 0$$

$$M = \left\{-4x^2 + 26x - 12\right\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$





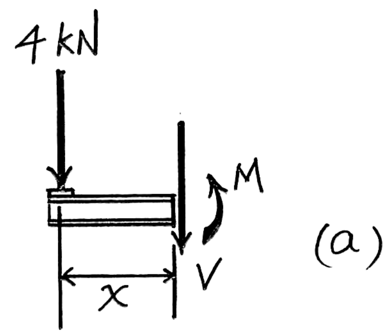
4–17. Determine the shear and moment throughout the beam as a function of x.



Internal Loadings. For $0 \le x \le 1$ m, referring to the FBD of the left segment of the beam in Fig. *a*,

$+\uparrow \sum F_y = 0;$ $\quad -V - 4 = 0 \quad V = -4$ kN **Ans.**

$\circlearrowleft + \sum M_O = 0;$ $\quad M + 4x = 0 \quad M = \{-4x\}$ kN·m **Ans.**

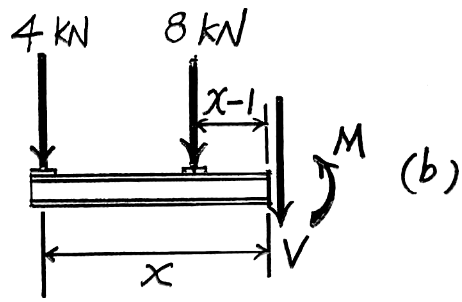


For $1 \text{ m} < x < 2$ m, referring to the FBD of the left segment of the beam in Fig. *b*,

$+\uparrow \sum F_y = 0;$ $\quad -4 - 8 - V = 0 \quad V = \{-12\}$ kN·m **Ans.**

$\circlearrowleft + \sum M_O = 0;$ $\quad M + 8(x - 1) + 4x = 0$

$M = \{-12x + 8\}$ kN·m **Ans.**

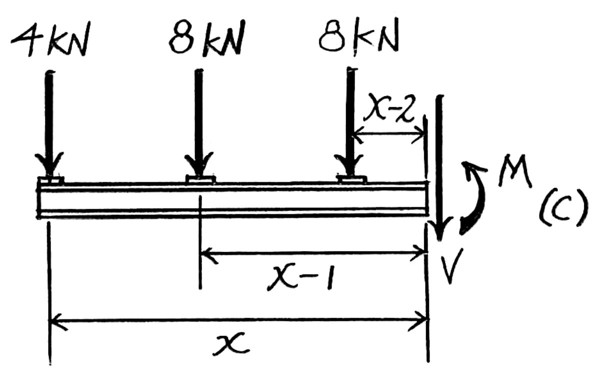


For $2 \text{ m} < x \le 3$ m, referring to the FBD of the left segment of the beam in Fig. *c*,

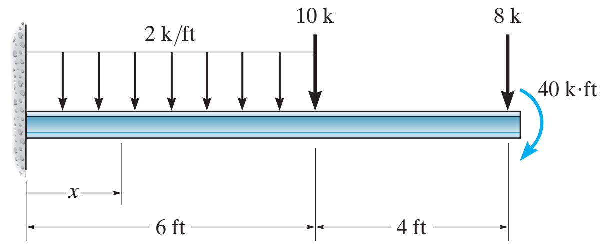
$+\uparrow \sum F_y = 0;$ $\quad -4 - 8 - 8 - V = 0 \quad V = \{-20\}$ kN **Ans.**

$\circlearrowleft + \sum M_O = 0;$ $\quad M + 4x + 8(x - 1) + 8(x - 2) = 0$

$M = \{-20x + 24\}$ kN·m **Ans.**

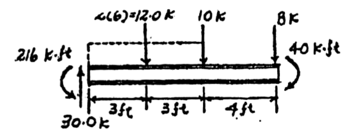


4–18. Determine the shear and moment throughout the beam as functions of x.



Support Reactions: As shown on FBD.

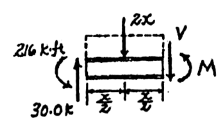
Shear and Moment Functions:



For $0 \leq x < 6$ ft

$$+\uparrow \sum F_y = 0; \quad 30.0 - 2x - V = 0$$

$$V = \{30.0 - 2x\} \text{ k}$$ **Ans.**

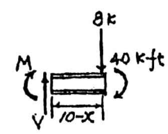


$$\circlearrowleft + \sum M_{NA} = 0; \quad M + 216 + 2x\left(\frac{x}{2}\right) - 30.0x = 0$$

$$M = \{-x^2 + 30.0x - 216\} \text{ k}\cdot\text{ft}$$ **Ans.**

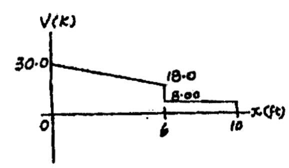
For $6 \text{ ft} < x \leq 10$ ft

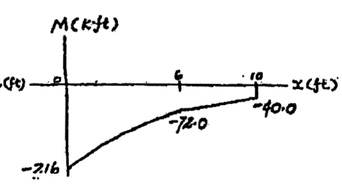
$$\overset{+}{\rightarrow} \sum F_y = 0; \quad V - 8 = 0 \qquad V = 8.00 \text{ k}$$ **Ans.**



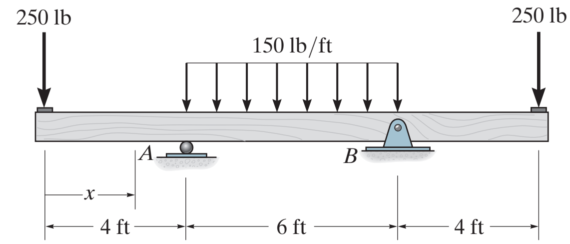
$$\circlearrowleft + \sum M_{NA} = 0; \quad -M - 8\,(10 - x) - 40 = 0$$

$$M = \{8.00x - 120\} \text{ k}\cdot\text{ft}$$ **Ans.**



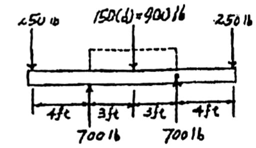


4–19. Determine the shear and moment throughout the beam as functions of x.



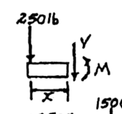
Support Reactions: As shown on FBD.

Shear and moment Functions:

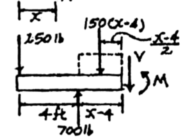


For $0 \le x < 4$ ft

$+\uparrow \sum F_y = 0; \quad -250 - V = 0 \qquad V = -250 \text{ lb}$ **Ans.**



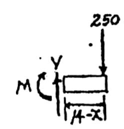
$\zeta + \sum M_{NA} = 0; \quad M + 250x = 0 \qquad M = \{-250x\} \text{ lb} \cdot \text{ft}$ **Ans.**



For 4 ft $< x <$ 10 ft

$+\uparrow \sum F_y = 0; \quad -250 + 700 - 150(x - 4) - V = 0$

$V = \{1050 - 150x\} \text{ lb}$ **Ans.**



$\zeta + \sum M_{NA} = 0; \quad M + 150(x - 4)\left(\frac{x - 4}{2}\right) + 250x - 700(x - 4) = 0$

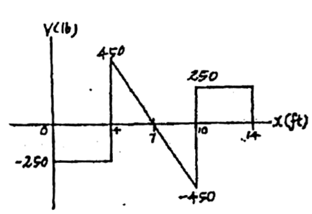
$M = \{-75x^2 + 1050x - 4000\} \text{lb} \cdot \text{ft}$ **Ans.**

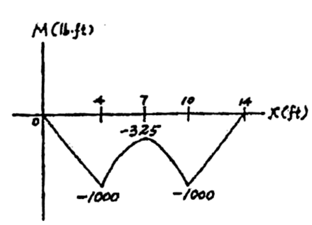
For 10 ft $< x \le$ 14 ft

$+\uparrow \sum F_y = 0; \quad V - 250 = 0 \qquad V = 250 \text{ lb}$ **Ans.**

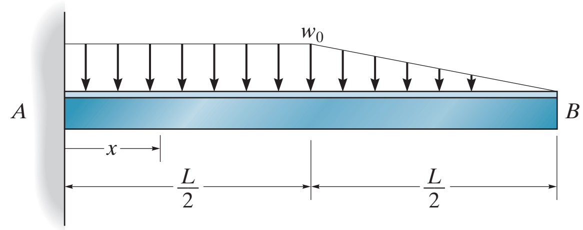
$\zeta + \sum M_{NA} = 0; \quad -M - 250(14 - x) = 0$

$M = \{250x - 3500\} \text{lb} \cdot \text{ft}$ **Ans.**



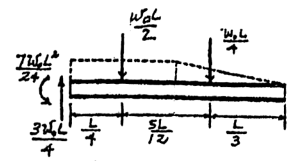


***4–20.** Determine the shear and moment in the beam as functions of x.



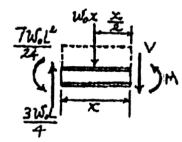
Support Reactions: As shown on FBD.

Shear and Moment Functions:



For $0 \le x < L/2$

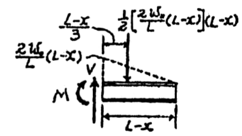
$+\uparrow \sum F_y = 0; \quad \frac{3w_oL}{4} - w_o x - V = 0 \qquad V = \frac{w_o}{4}(3L - 4x)$ **Ans.**



$\circlearrowleft + \sum M_{NA} = 0; \quad \frac{7w_oL^2}{24} - \frac{3w_oL}{4}x + w_o x\left(\frac{x}{2}\right) + M = 0$

$M = \frac{w_o}{24}(-12x^2 + 18Lx - 7L^2)$ **Ans.**

For $L/2 < x \le L$

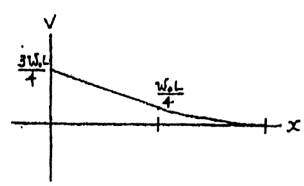


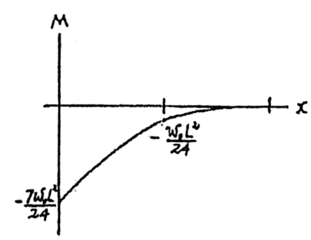
$+\uparrow \sum F_y = 0; \quad V - \frac{1}{2}\left[\frac{2w_o}{L}(L-x)\right](L-x) = 0$

$V = \frac{w_o}{L}(L-x)^2$ **Ans.**

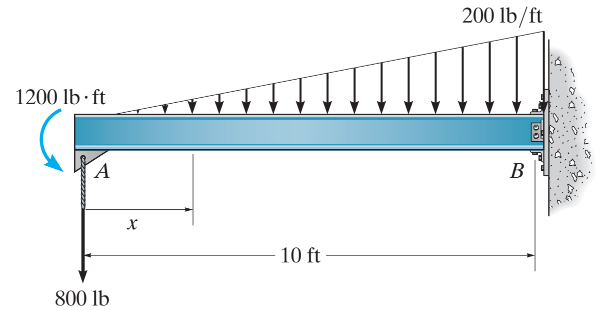
$\circlearrowleft + \sum M_{NA} = 0; \quad -M - \frac{1}{2}\left[\frac{2w_o}{L}(L-x)\right](L-x)\left(\frac{L-x}{3}\right) = 0$

$M = \frac{w_o}{3L}(L-x)^2$ **Ans.**





4–21. Determine the shear and moment in the beam as a function of x.



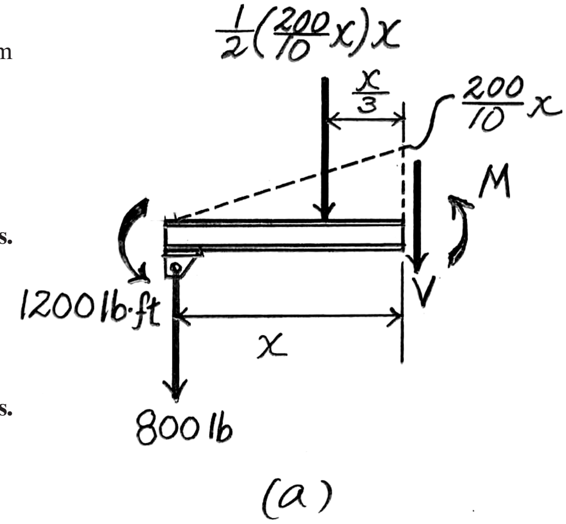
Internal Loadings: Referring to the FBD of the left segment of the beam in Fig. *a*,

$$+\uparrow \sum F_y = 0; \quad -800 - \frac{1}{2}\left(\frac{200}{10}x\right)(x) - V = 0$$

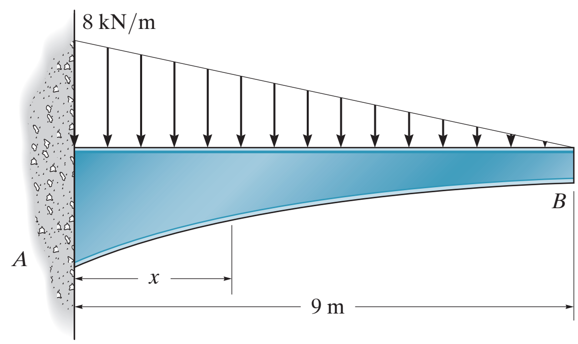
$$V = \{-10x^2 - 800\}\text{lb} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_o = 0; \quad M + \frac{1}{2}\left(\frac{200}{10}x\right)(x)\left(\frac{x}{3}\right) + 800x + 1200 = 0$$

$$M = \{-3.33x^3 - 800x - 1200\}\ \text{lb} \cdot \text{ft} \qquad \textbf{Ans.}$$



4–22 Determine the shear and moment throughout the tapered beam as a function of x.



$$\overset{+}{\rightarrow} \sum F_y = 0; \quad 36 - \frac{1}{2}\left(\frac{8}{9}x\right)(x) - \frac{8}{9}\left(8 - \frac{8}{9}x\right)x - V = 0$$

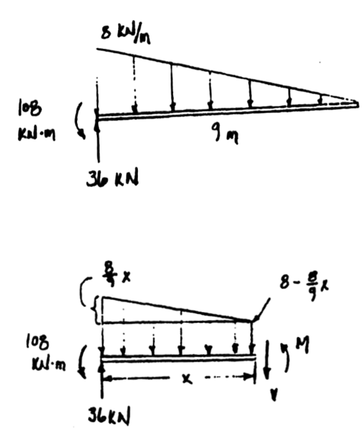
$$V = 36 - \frac{4}{9}x^2 - 8x + \frac{8}{9}x^2$$

$$V = 0.444x^2 - 8x + 36$$

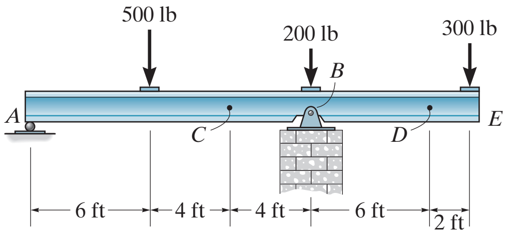
$$\circlearrowleft + \sum M = 0; \quad 108 + \frac{1}{2}\left(\frac{8}{9}x\right)(x)\left(\frac{2}{3}x\right) + \frac{8}{9}\left(8 - \frac{8}{9}x\right)(x)\left(\frac{x}{2}\right) - 36x + M = 0$$

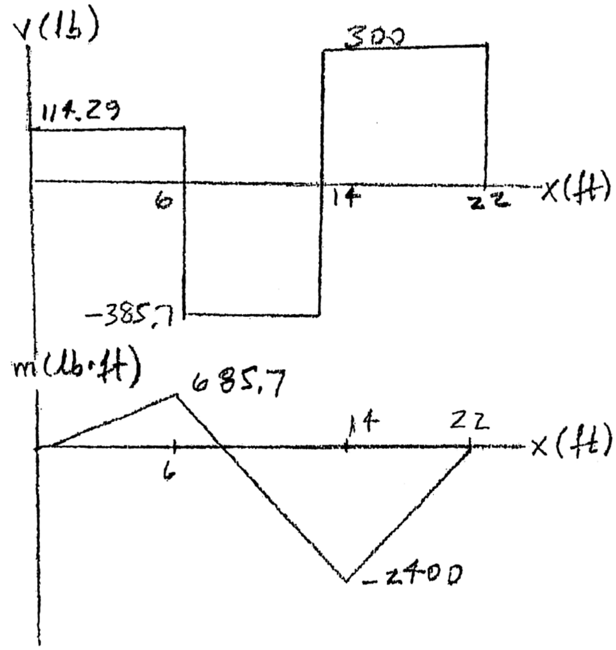
$$M = -108 - \frac{8}{27}x^3 - 4x^2 + \frac{8}{18}x^3 + 36x$$

$$M = 0.148x^3 - 4x^2 + 36x - 108 \qquad \textbf{Ans.}$$

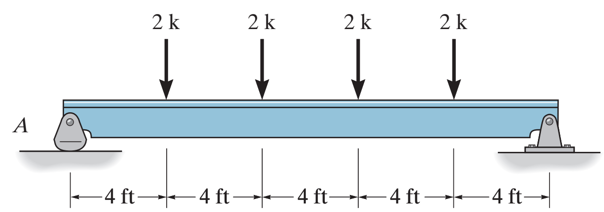


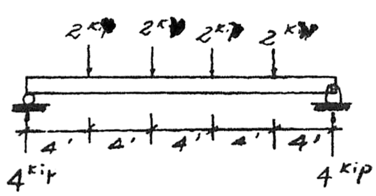
4–23. Draw the shear and moment diagrams for the beam.

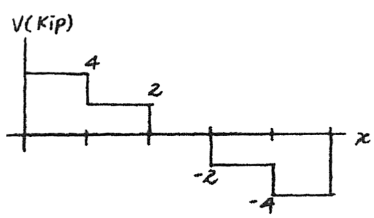


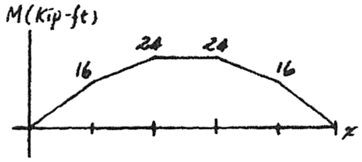


***4–24.** Draw the shear and moment diagrams for the beam.

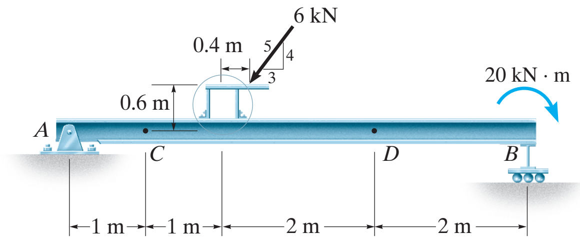






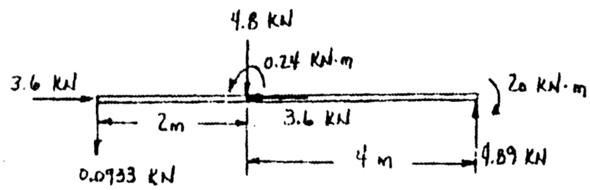


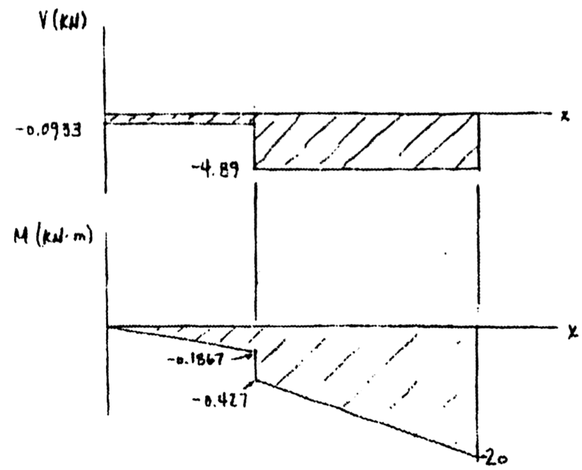
4–25. Draw the shear and moment diagrams for the beam.



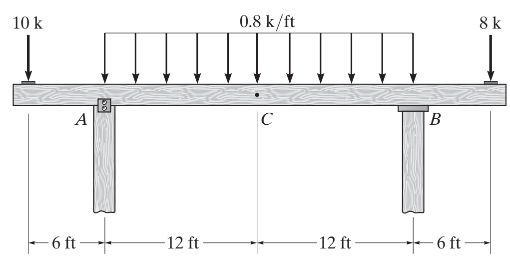
$V_{\max} = -4.89$ kN **Ans.**

$M_{\max} = -20$ kN · m **Ans.**



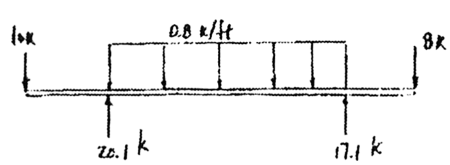


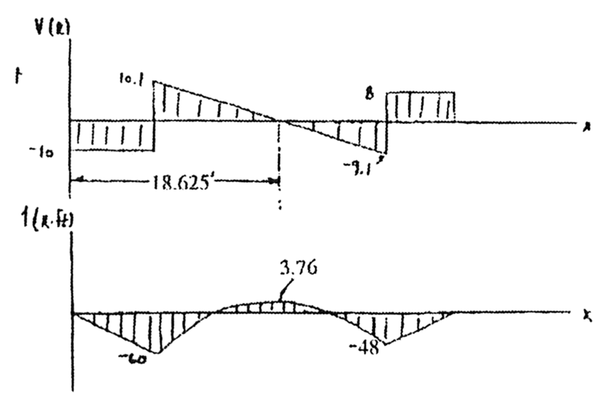
4–26. Draw the shear and moment diagrams of the beam.



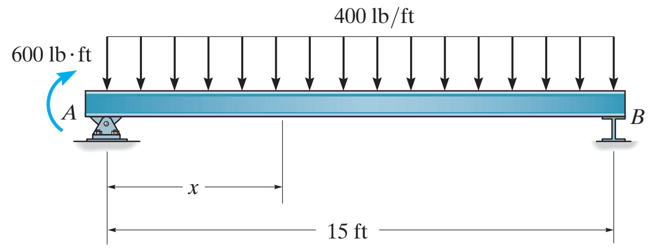
$V_{\max} = -10.1$ k **Ans.**

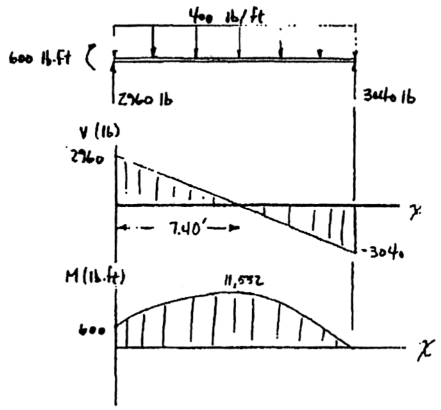
$M_{\max} = -60$ k · ft **Ans.**



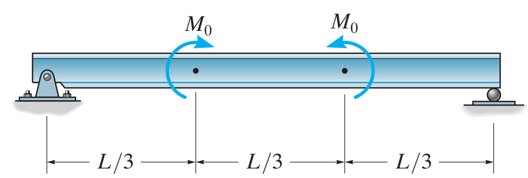


4–27. Draw the shear and moment diagrams for the beam.

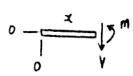




***4–28.** Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set M_O = 500 N · m, L = 8 m.

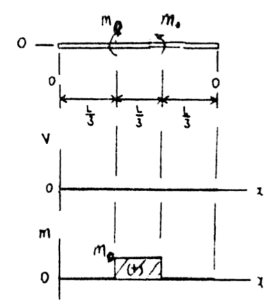


(a) For $0 \leq x \leq \dfrac{L}{3}$

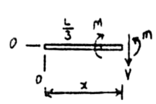


$+\uparrow \sum F_y = 0; \quad V = 0$ **Ans.**

$\circlearrowleft + \sum M = 0; \quad M = 0$ **Ans.**



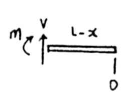
For $\dfrac{L}{3} < x < \dfrac{2L}{3}$



$+\uparrow \sum F_y = 0; \quad V = 0$ **Ans.**

$\circlearrowleft + \sum M = 0; \quad M = M_O$ **Ans.**

For $\dfrac{2L}{3} < x \leq L$

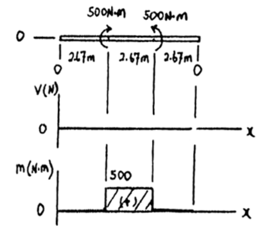


$+\uparrow \sum F_y = 0; \quad V = 0$ **Ans.**

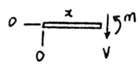
$\circlearrowleft + \sum M = 0; \quad M = 0$ **Ans.**

4–28. Continued

(b) Set $M_O = 500\ \text{N}\cdot\text{m},\ L = 8\ \text{m}$



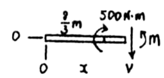
For $0 \le x < \frac{8}{3}$ m



$+\uparrow \sum F_y = 0;\quad V = 0$ **Ans.**

$\circlearrowleft + \sum M = 0;\quad M = 0$ **Ans.**

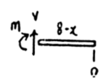
For $\frac{8}{3}\text{ m} < x < \frac{16}{3}\text{ m}$



$+\uparrow \sum F_y = 0;\quad V = 0$ **Ans.**

$\circlearrowleft + \sum M = 0;\quad M = 500\ \text{N}\cdot\text{m}$ **Ans.**

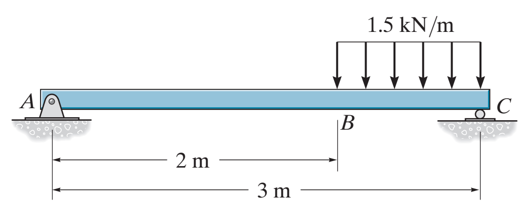
For $\frac{16}{3}\text{ m} < x \le 8\text{ m}$



$+\uparrow \sum F_y = 0;\quad V = 0$ **Ans.**

$\circlearrowleft + \sum M = 0;\quad M = 0$ **Ans.**

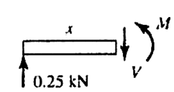
4–29. Draw the shear and moment diagrams for the beam.



Support Reactions:

$\circlearrowleft + \sum M_A = 0;\quad C_x(3) - 1.5(2.5) = 0 \quad C_x = 1.25\ \text{kN}$

$+\uparrow \sum F_y = 0;\quad A_y - 1.5 + 1.25 = 0 \quad A_y = 0.250\ \text{kN}$

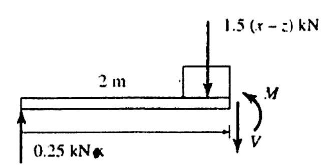


(a)

Shear and Moment Functions: For $0 \le x < 2$ m [FBD (a)],

$+\uparrow \sum F_y = 0;\quad 0.250 - V = 0 \quad V = 0.250\ \text{kN}$ **Ans.**

$\circlearrowleft + \sum M = 0;\quad M - 0.250x = 0 \quad M = (0.250x)\ \text{kN}\cdot\text{m}$ **Ans.**



(b)

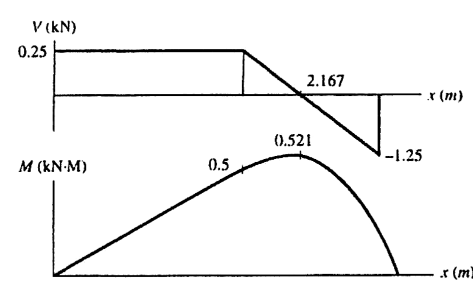
For $2\text{ m} < x \le 3$ m [FBD (b)].

$+\uparrow \sum F_y = 0;\quad 0.25 - 1.5(x - 2) - V = 0$

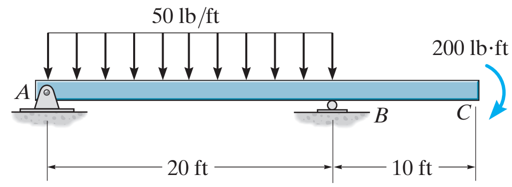
$V = (3.25 - 1.50x)\ \text{kN}$ **Ans.**

$\circlearrowleft + \sum M = 0;\quad -0.25x + 1.5(x + 2)\left(\frac{x-2}{2}\right) + M = 0$

$M = (-0.750x^2 + 3.25x - 3.00)\ \text{kN}\cdot\text{m}$ **Ans.**

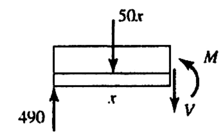


4–30. Draw the shear and bending-moment diagrams for the beam.



Support Reactions:

$\circlearrowleft + \sum M_B = 0;\quad 1000(10) - 200 - A_y(20) = 0 \quad A_y = 490 \text{ lb}$



(a)

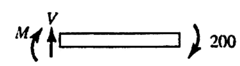
Shear and Moment Functions: For $0 \le x < 20$ ft [FBD (a)].

$+\uparrow \sum F_y = 0;\quad 490 - 50x - V = 0$

$V = \{490 - 50.0x\} \text{ lb}$ **Ans.**

$\circlearrowleft + \sum M = 0;\quad M + 50x\left(\frac{x}{2}\right) - 490x = 0$

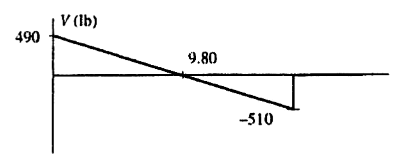
$M = (490x - 25.0x^2) \text{ lb} \cdot \text{ft}$ **Ans.**

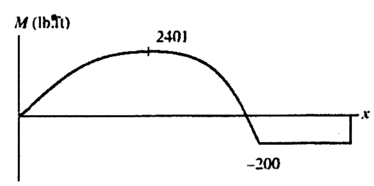


For $20 \text{ ft} < x \le 30$ ft [FBD (b)],

$+\uparrow \sum F_y = 0;\quad V = 0$ **Ans.**

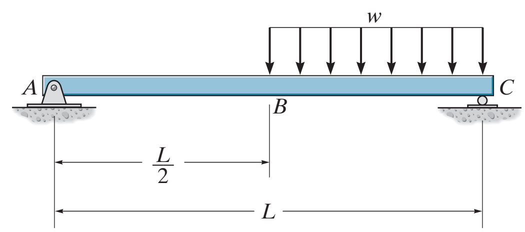
$\circlearrowleft + \sum M = 0;\quad -200 - M = 0 \quad M = -200 \text{ lb} \cdot \text{ft}$ **Ans.**



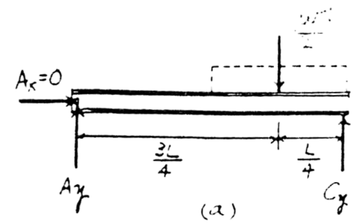


(b)

4–31. Draw the shear and moment diagrams for the beam.



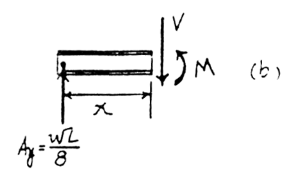
Support Reactions: From FBD(a),



$$\zeta + \sum M_A = 0; \quad C_y(L) - \frac{wL}{2}\left(\frac{3L}{4}\right) = 0 \quad C_y = \frac{3wL}{8}$$

$$+\uparrow \sum F_y = 0; \quad A_y + \frac{3wL}{8} - \left(\frac{wL}{2}\right) = 0 \quad A_y = \frac{wL}{8}$$

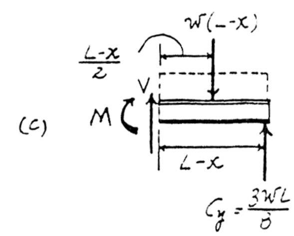
Shear and Moment Functions: For $0 \le x < \frac{L}{2}$ [FBD (b)],



$$+\uparrow \sum F_y = 0; \quad \frac{wL}{8} - V = 0 \quad V = \frac{wL}{8}$$ **Ans.**

$$\zeta + \sum M = 0; \quad M - \frac{wL}{8}(x) = 0 \quad M = \frac{wL}{8}(x)$$ **Ans.**

For $\frac{L}{2} < x \le L$ [FBD (c)],

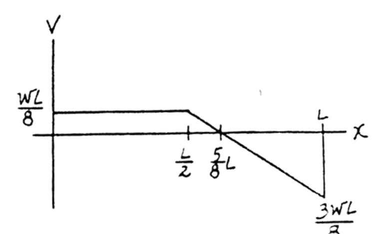


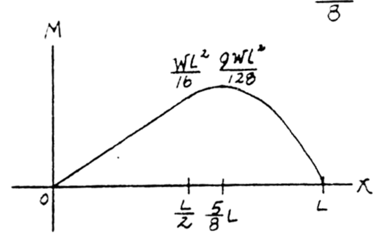
$$+\uparrow \sum F_y = 0; \quad V + \frac{3wL}{8} - w(L - x) = 0$$

$$V = \frac{w}{8}(5L - 8x)$$

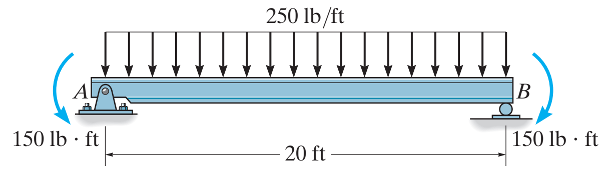
$$\zeta + \sum M_B = 0; \quad \frac{3wL}{8}(L - x) - w(L - x)\left(\frac{L - x}{2}\right) - M = 0$$

$$M = \frac{w}{8}(-L^2 + 5Lx - 4x^2)$$ **Ans.**



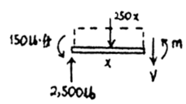


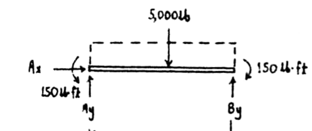
***4–32.** Draw the shear and moment diagrams for the beam.



$\circlearrowleft + \sum M_A = 0;\quad -5000(10) - 150 + B_y(20) = 0$

$B_y = 2500 \text{ lb}$

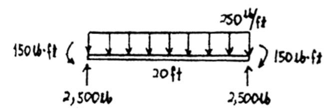




$\overset{+}{\rightarrow} \sum F_x = 0;\quad A_x = 0$

$+\uparrow \sum F_y = 0;\quad A_y = 5000 + 2500 = 0$

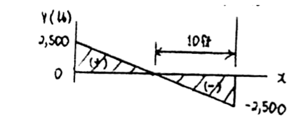
$A_y = 2500 \text{ lb}$



For $0 \le x \le 20$ ft

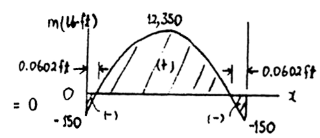
$+\uparrow \sum F_y = 0;\quad 2500 - 250x - V = 0$

$V = 250(10 - x)$ **Ans.**

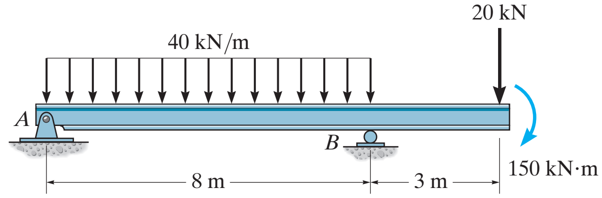


$\circlearrowleft + \sum M = 0;\quad -2500(x) + 150 + 250x\left(\frac{x}{2}\right) + M = 0$

$M = 25(100x - 5x^2 - 6)$ **Ans.**



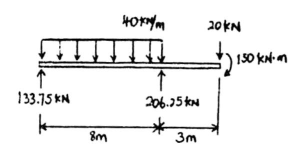
4–33. Draw the shear and moment diagrams for the beam.



$0 \le x < 8$

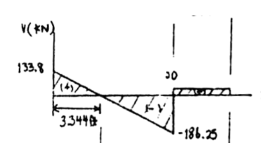
$+\uparrow \sum F_y = 0;\quad 133.75 - 40x - V = 0$

$V = 133.75 - 40x$ **Ans.**



$\circlearrowleft + \sum M = 0;\quad M + 40x\left(\frac{x}{2}\right) - 133.75x = 0$

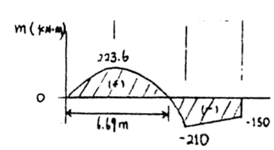
$M = 133.75x - 20x^2$ **Ans.**



$8 < x \le 11$

$+\uparrow \sum F_y = 0;\quad V - 20 = 0$

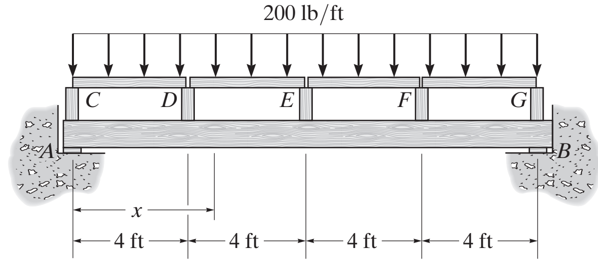
$V = 20$ **Ans.**



$\circlearrowright + \sum M = 0;\quad M + 20(11 - x) + 150 = 0$

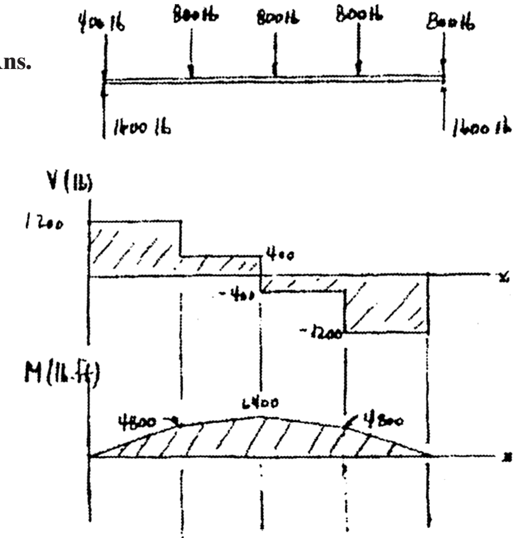
$M = 20x - 370$ **Ans.**

4–34. Draw the shear and moment diagrams for the beam.

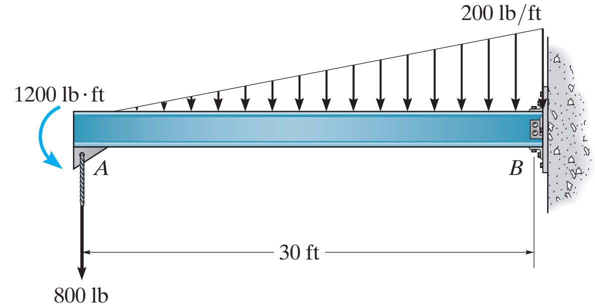


$V_{max} = \pm 1200$ lb **Ans.**

$M_{max} = 6400$ lb · ft **Ans.**

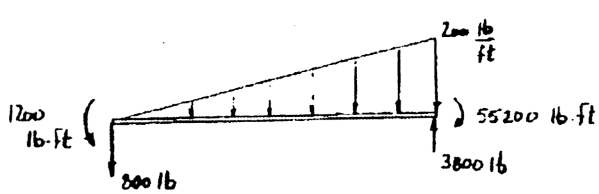


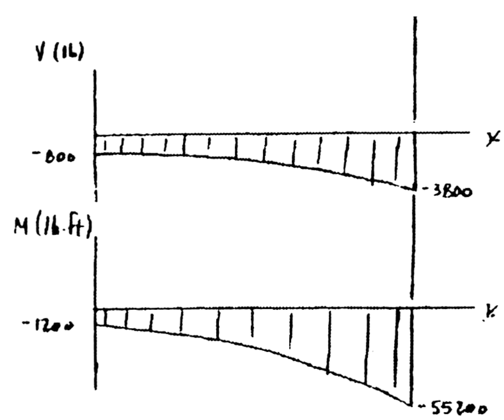
4–35. Draw the shear and moment diagrams for the beam.



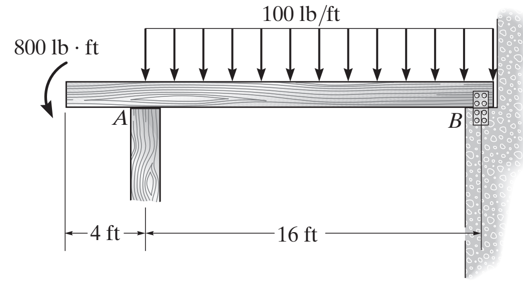
$V_{max} = -3.80$ k **Ans.**

$M_{max} = -55.2$ k · ft **Ans.**



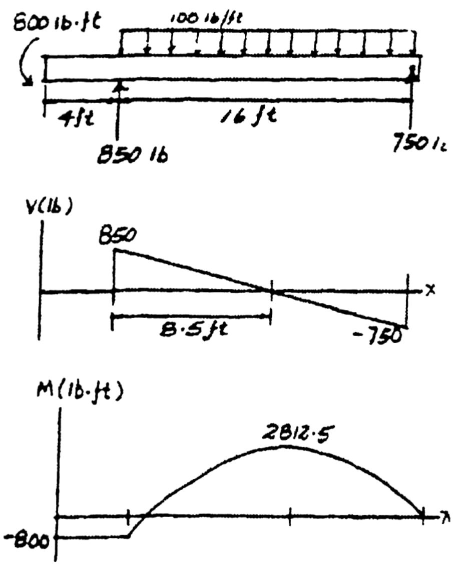


***4–36.** Draw the shear and moment diagrams of the beam. Assume the support at B is a pin and A is a roller.

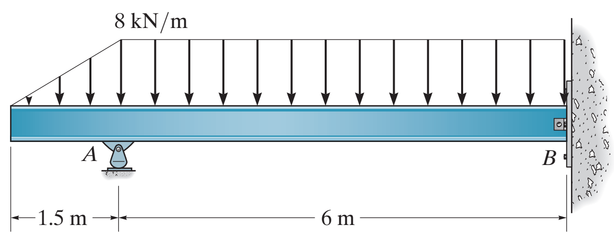


$V_{max} = 850$ lb **Ans.**

$M_{max} = -2.81$ K · ft **Ans.**

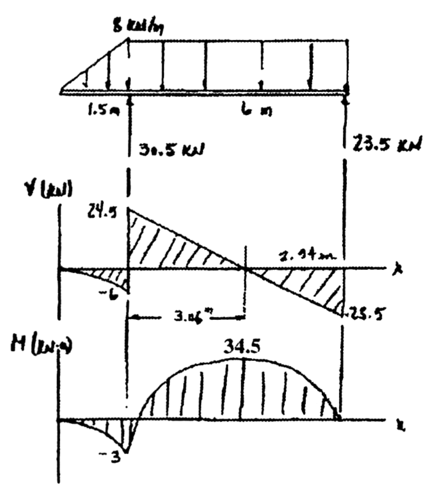


4–37. Draw the shear and moment diagrams for the beam. Assume the support at B is a pin.

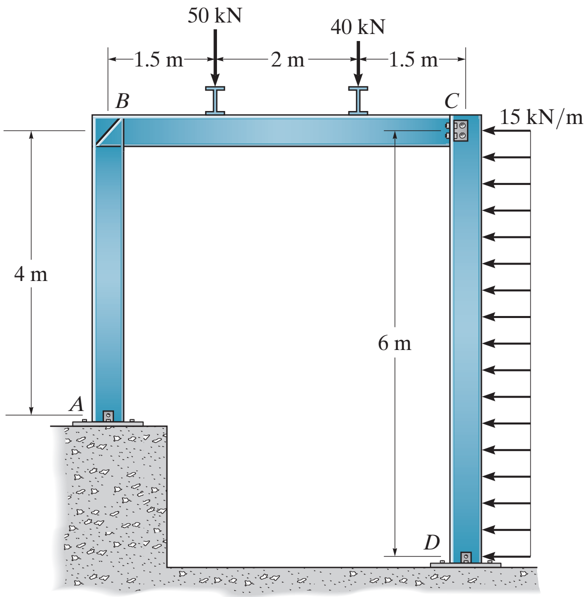


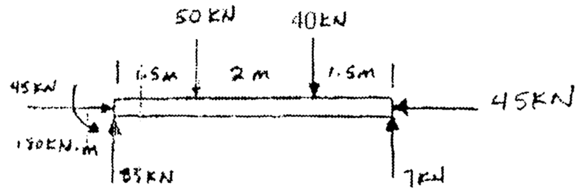
$V_{max} = 24.5$ kN **Ans.**

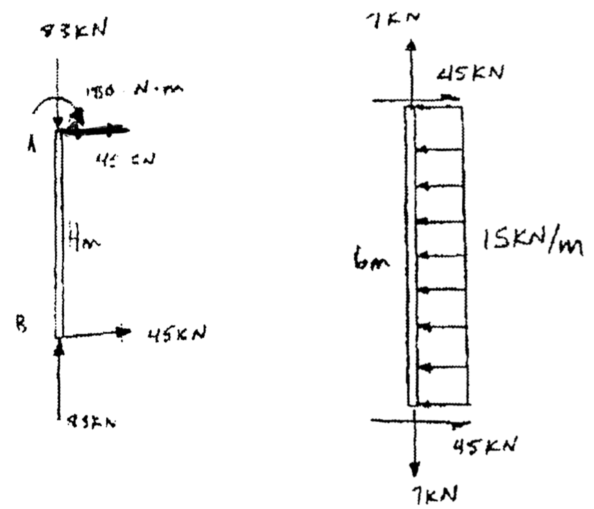
$M_{max} = 34.5$ kN · m **Ans.**

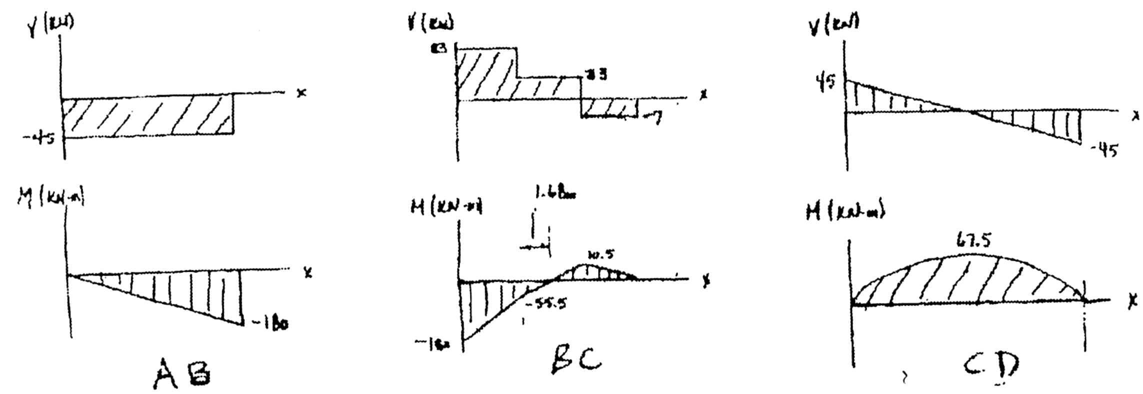


4–38. Draw the shear and moment diagrams for each of the three members of the frame. Assume the frame is pin connected at A, C, and D and there is fixed joint at B.

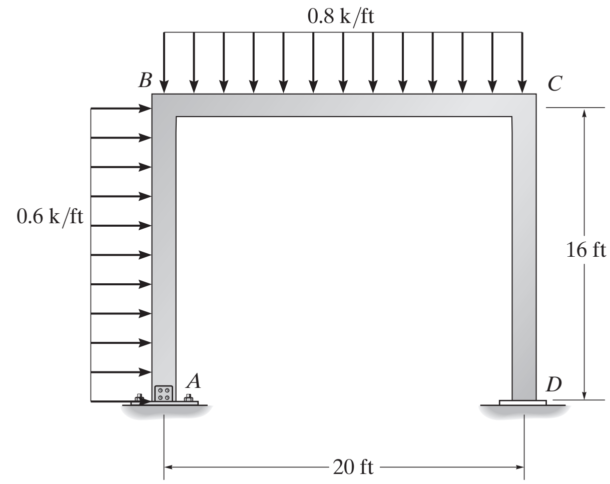






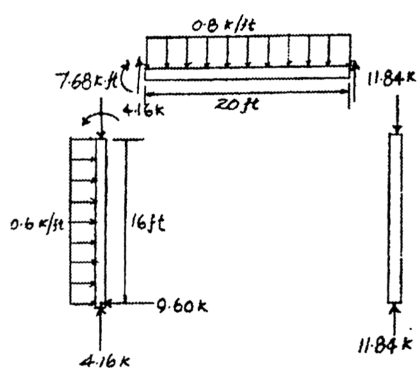


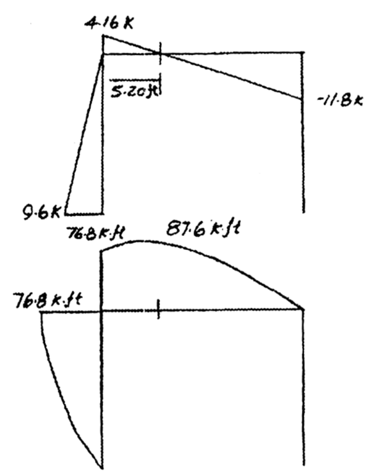
4–39. Draw the shear and moment diagrams for each member of the frame. Assume the support at A is a pin and D is a roller.



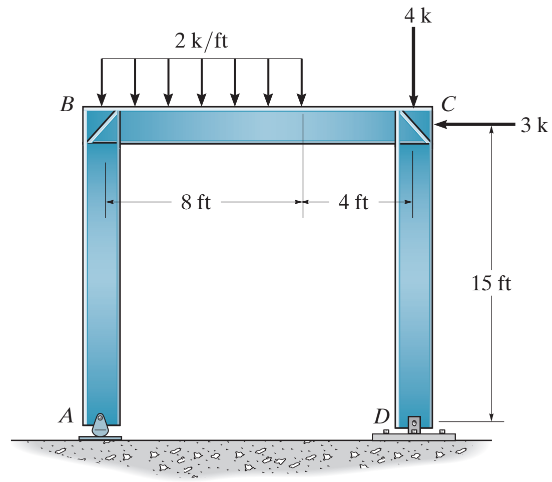
$V_{\max} = -11.8 \text{ k}$ **Ans.**

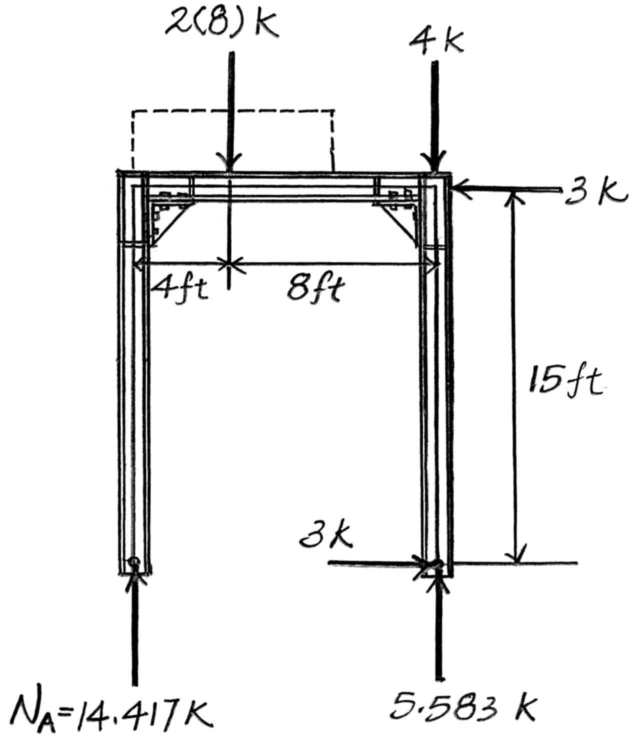
$M_{\max} = -87.6 \text{ k} \cdot \text{ft}$ **Ans.**

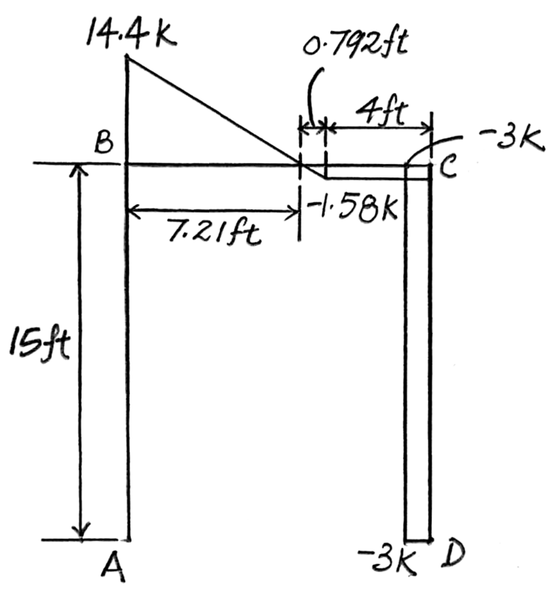


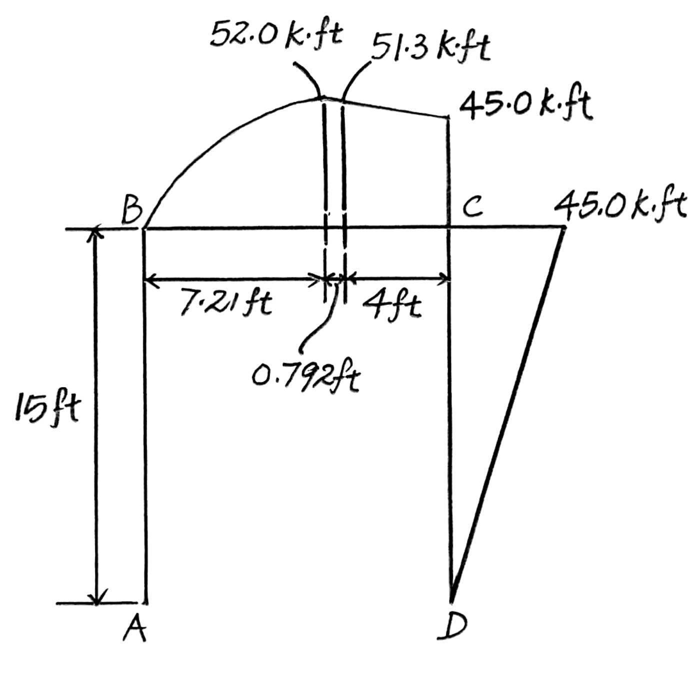


***4–40.** Draw the shear and moment diagrams for each member of the frame. Assume A is a rocker, and D is pinned.

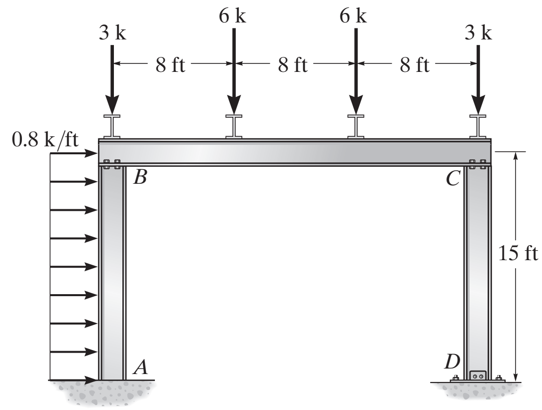


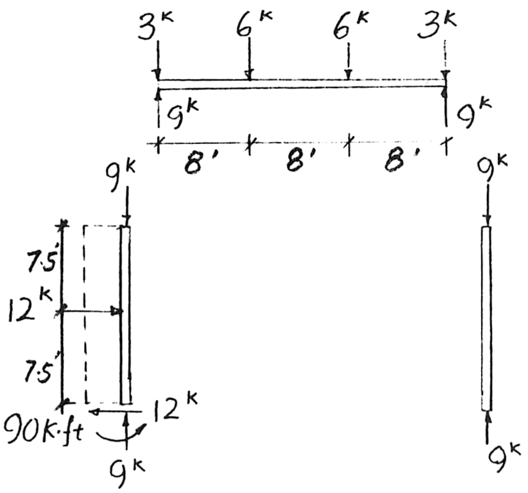




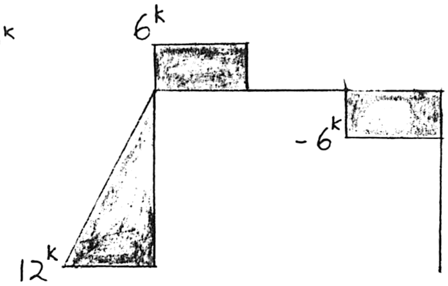


4–41. Draw the shear and moment diagrams for each member of the frame. Assume the frame is pin connected at B, C, and D and A is fixed.

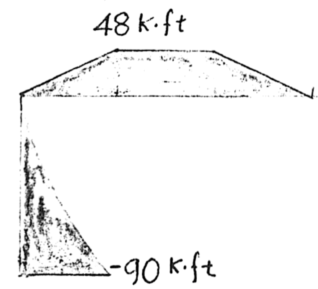




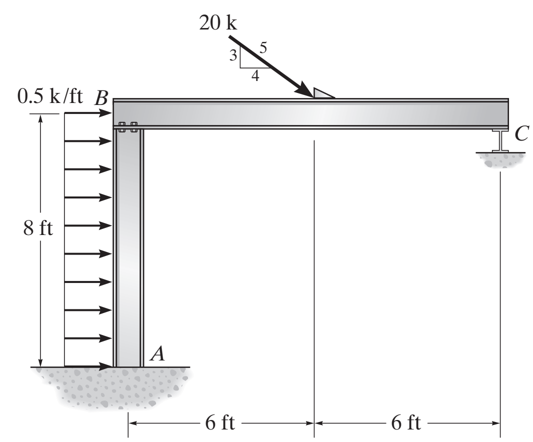
Shear diagram



moment diagram

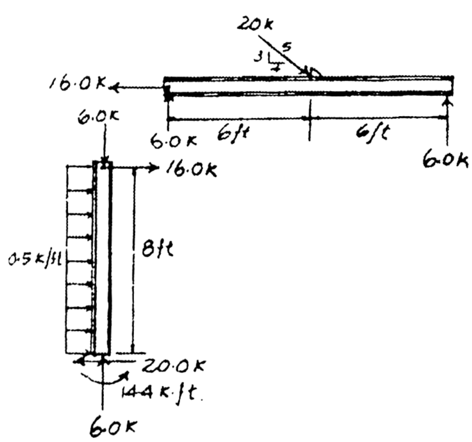


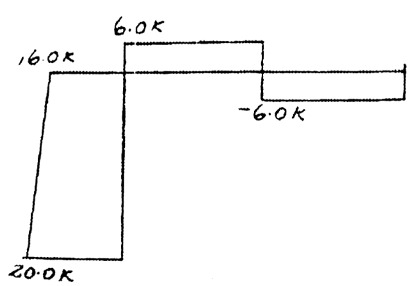
4–42. Draw the shear and moment diagrams for each member of the frame. Assume A is fixed, the joint at B is a pin, and support C is a roller.

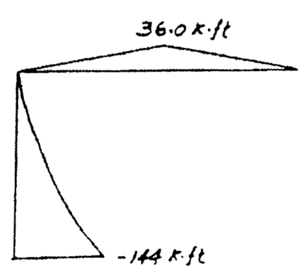


$V_{max} = 20.0 \text{ k}$ **Ans.**

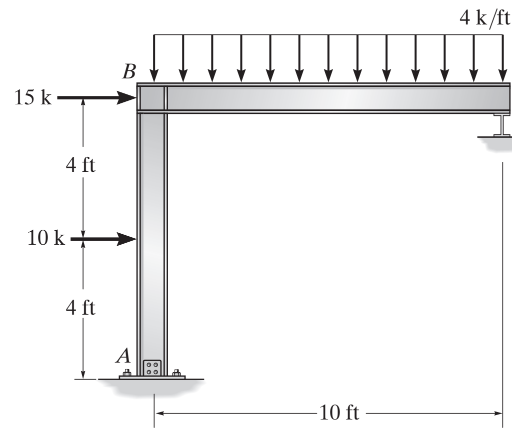
$M_{max} = -144 \text{ k} \cdot \text{ft}$ **Ans.**

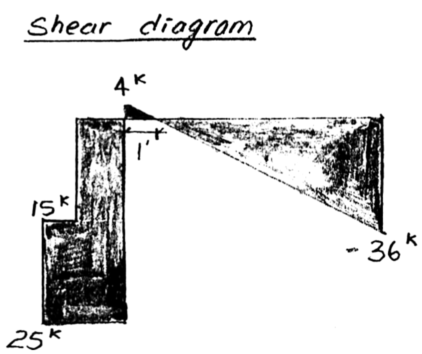


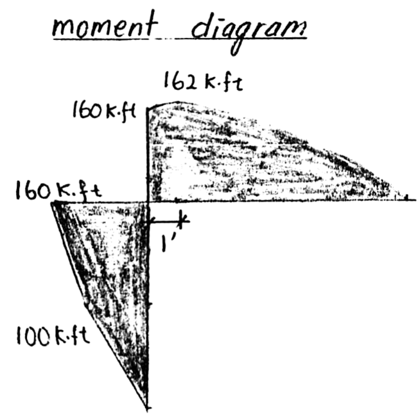


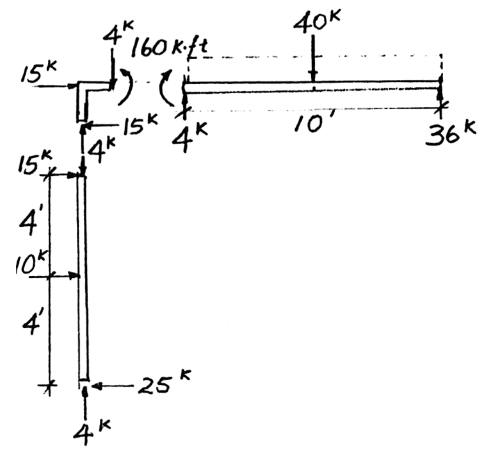


4–43. Draw the shear and moment diagrams for each member of the frame. Assume the frame is pin connected at *A*, and *C* is a roller.

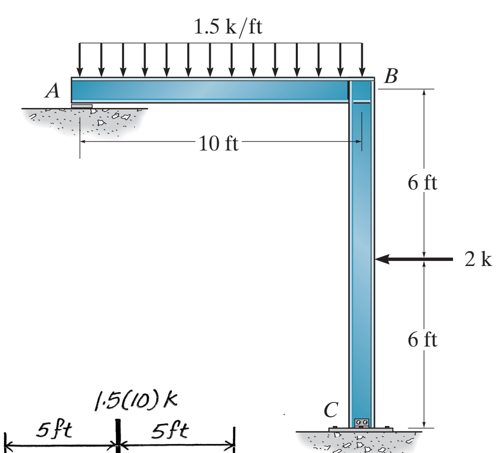


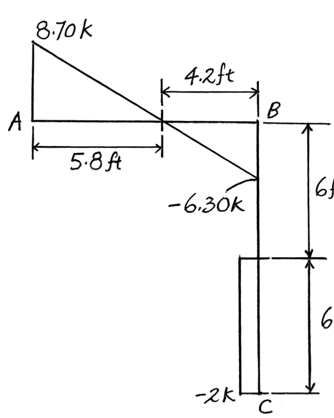


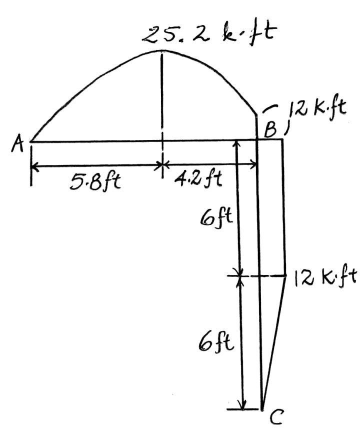


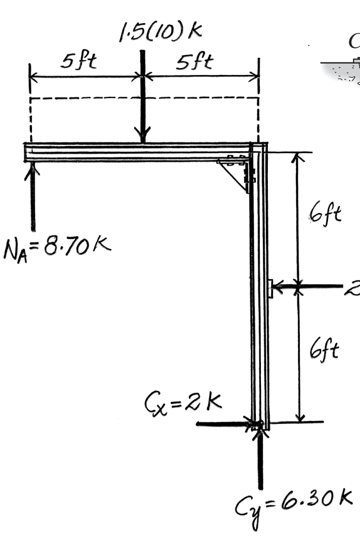


***4–44.** Draw the shear and moment diagrams for each member of the frame. Assume the frame is roller supported at *A* and pin supported at *C*.

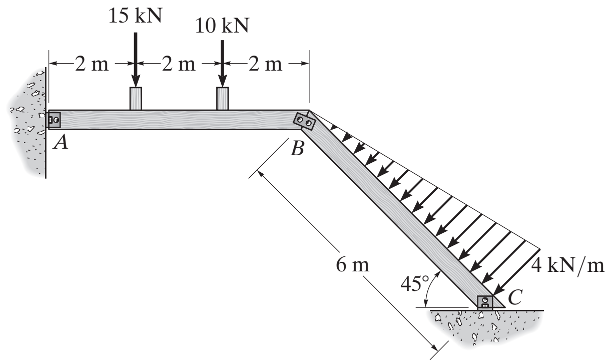








4–45. Draw the shear and moment diagrams for each member of the frame. The members are pin connected at A, B, and C.



Support Reactions:

$$\zeta+\sum M_A = 0;\quad -15(2) - 10(4) + B_y(6) = 0$$

$$B_y = 11.667 \text{ kN}$$

$$+\uparrow \sum F_y = 0;\quad A_y - 25 + 11.667 = 0$$

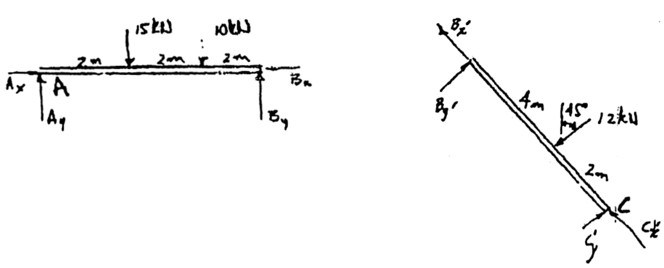
$$A_y = 13.3 \text{ kN}$$

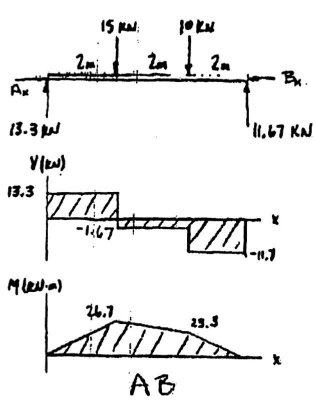
$$\zeta+\sum M_c = 0;\quad 12(2) - B_{y'}(6) = 0$$

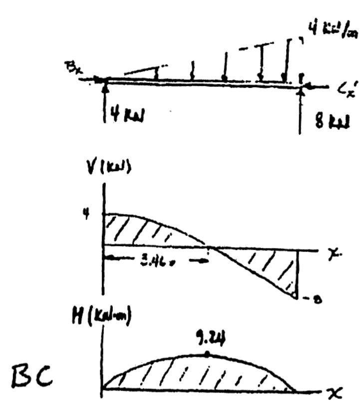
$$B_{y'} = 4 \text{ kN}$$

$$+\nearrow \sum F_{y'} = 0;\quad 4 - 12 + C_{y'} = 0$$

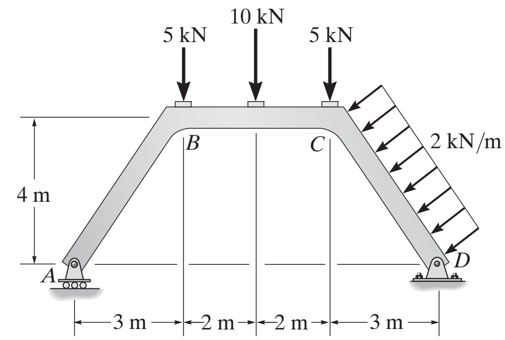
$$C_{y'} = 8 \text{ kN}$$







4–46. Draw the shear and moment diagrams for each member of the frame.



$$\circlearrowleft + \sum M_D = 0; \quad 10(2.5) + 5(3) + 10(5) + 5(7) - A_y(10) = 0$$

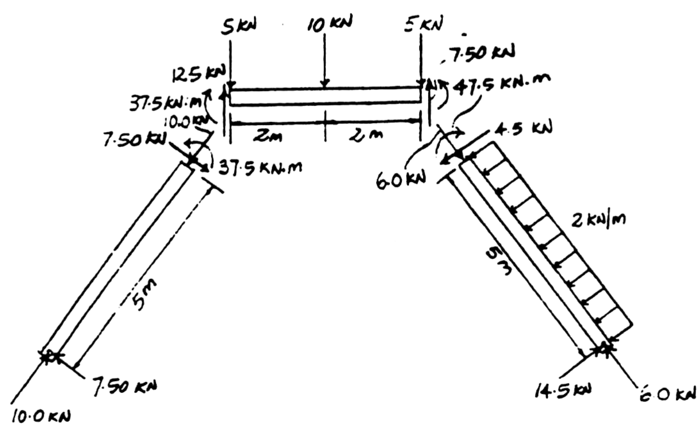
$$A_y = 12.5 \text{ kN}$$

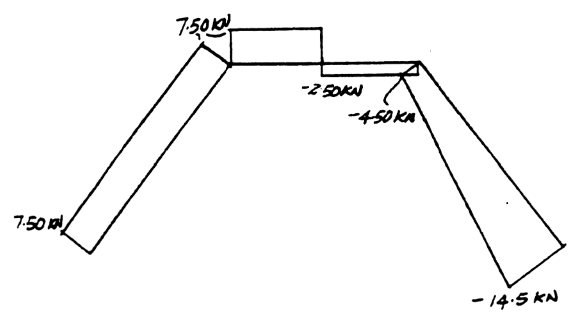
$$\xrightarrow{+} \sum F_x = 0; \quad -10\left(\frac{4}{5}\right) + D_x = 0$$

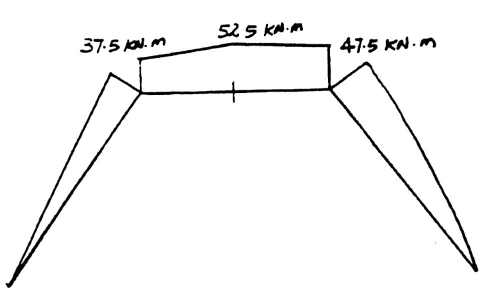
$$D_x = 8 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad 12.5 - 5 - 10 - 5 - 10\left(\frac{3}{5}\right) + D_y = 0$$

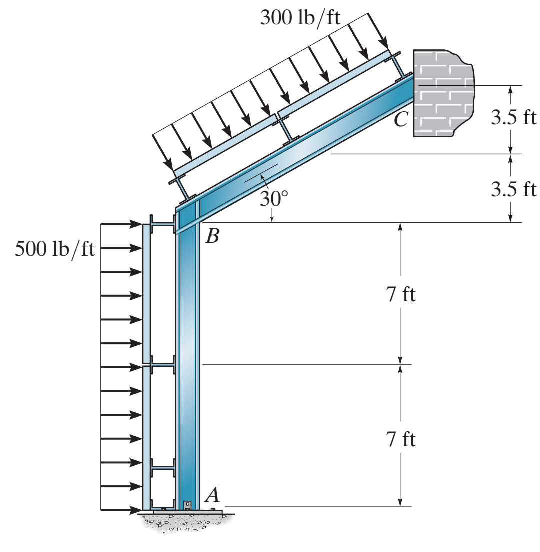
$$D_y = 13.5 \text{ kN}$$







4–47. Draw the shear and moment diagrams for each member of the frame. Assume the joint at A is a pin and support C is a roller. The joint at B is fixed. The wind load is transferred to the members at the girts and purlins from the simply supported wall and roof segments.



Support Reactions:

$$\zeta + \sum M_A = 0; \quad -3.5(7) - 1.75(14) - (4.20)(\sin 30°)(7\cos 30°)$$

$$-4.20(\sin 30°)(14 + 3.5) + (21) = 0$$

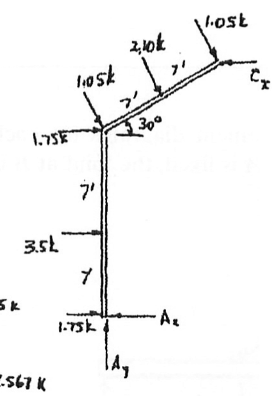
$$C_x = 5.133 \text{ kN}$$

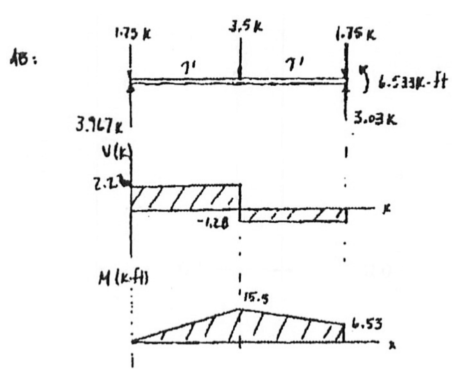
$$\xrightarrow{+} \sum F_x = 0; \quad 1.75 + 3.5 + 1.75 + 4.20 \sin 30° - 5.133 - A_x = 0$$

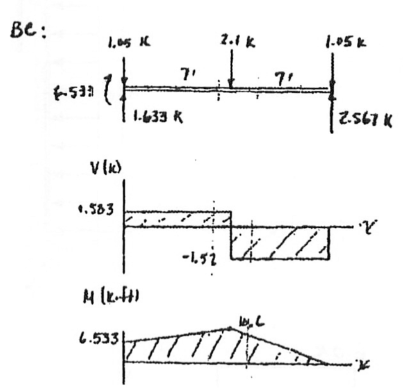
$$A_x = 3.967 \text{ kN}$$

$$+\uparrow \sum F_y = 0; \quad A_y - 4.20 \cos 30° = 0$$

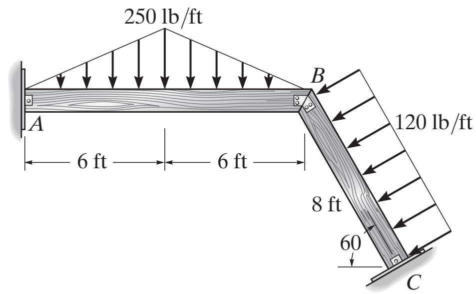
$$A_y = 3.64 \text{ kN}$$

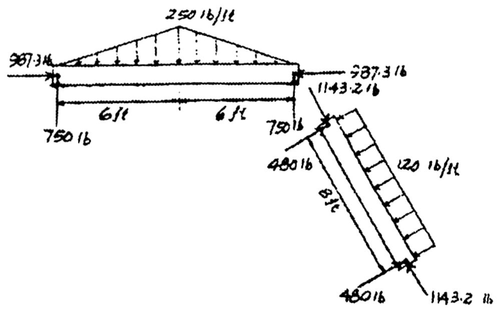


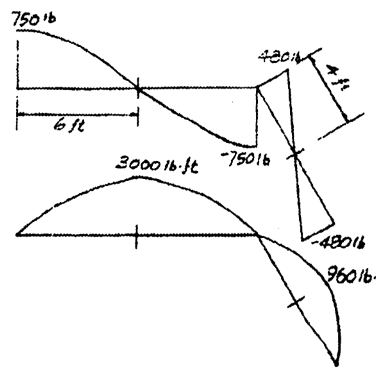




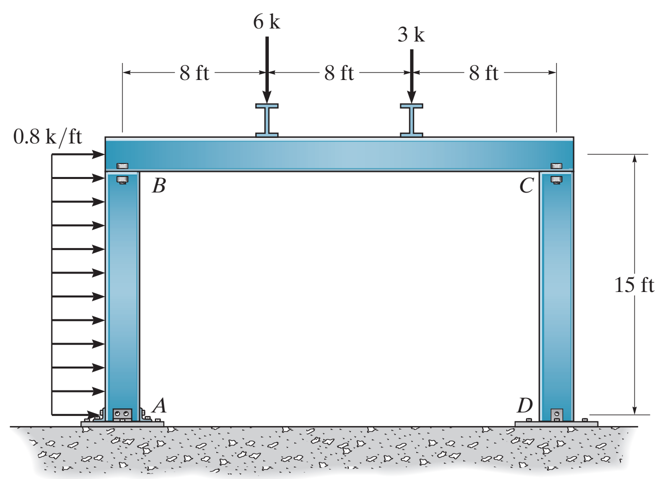
***4–48.** Draw the shear and moment diagrams for each member of the frame. The joints at A, B and C are pin connected.

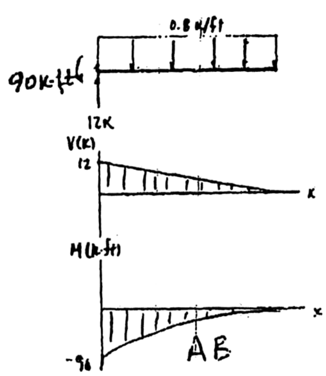


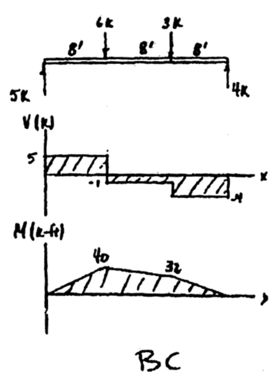


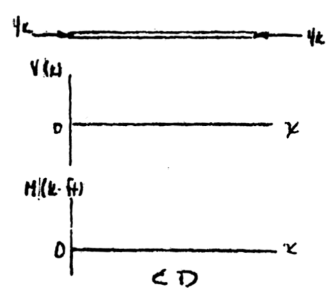


4–49. Draw the shear and moment diagrams for each of the three members of the frame. Assume the frame is pin connected at B, C and D and A is fixed.

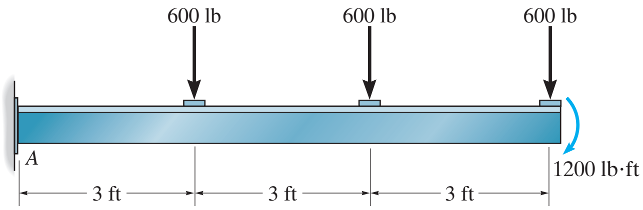


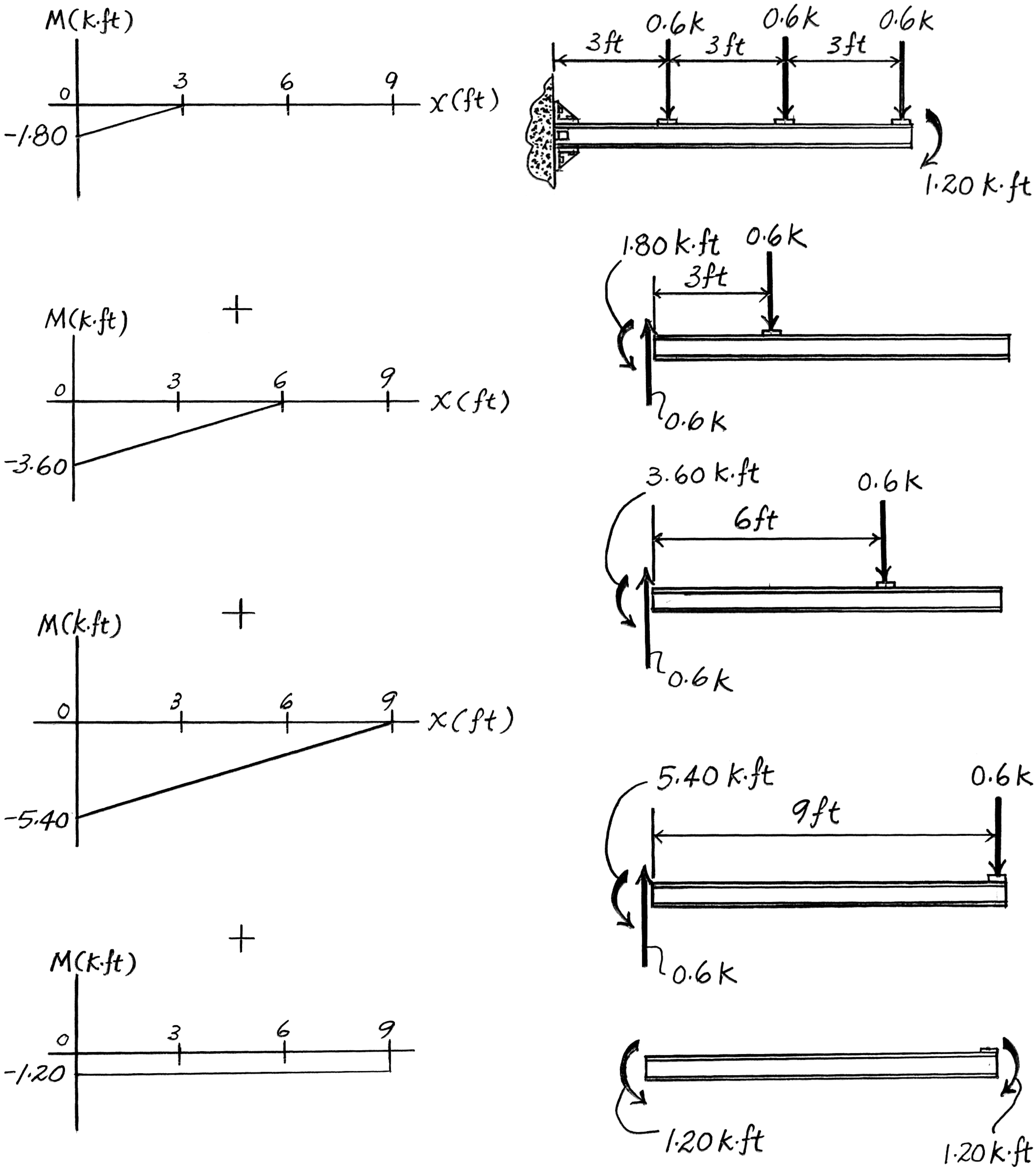




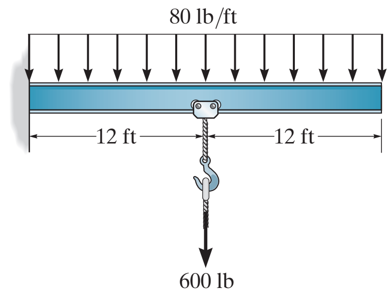


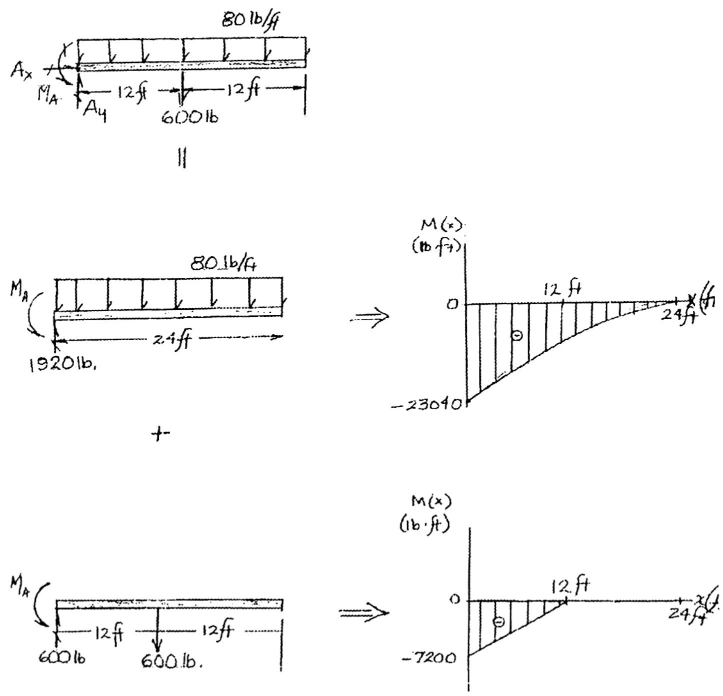
4–50. Draw the moment diagrams for the beam using the method of superposition. The beam is cantilevered from *A*.



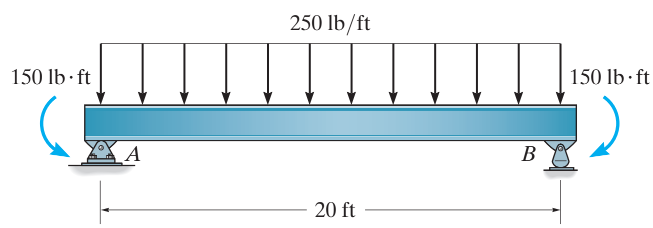


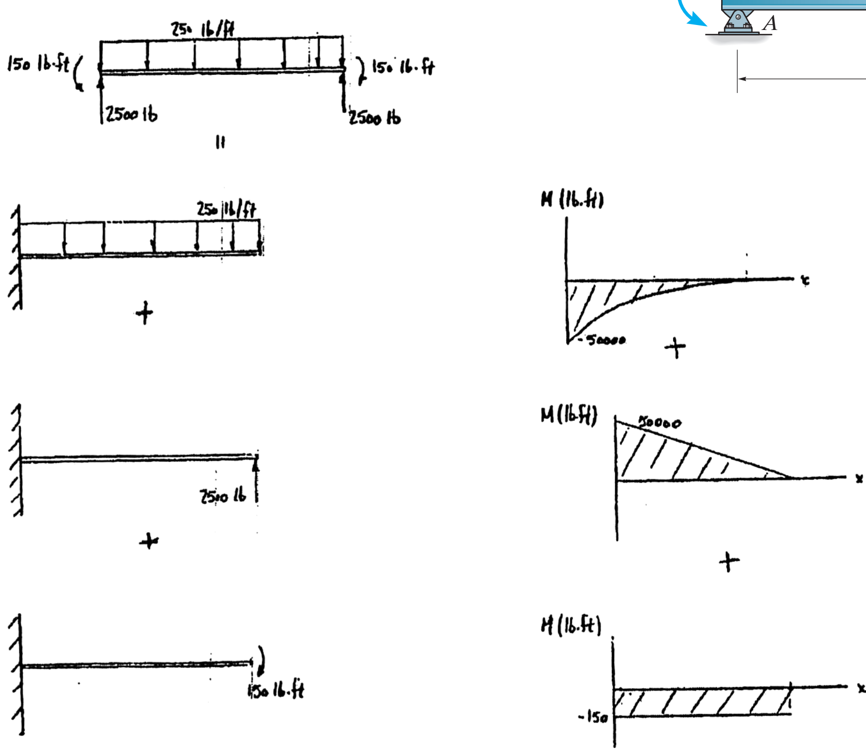
4–51. Draw the moment diagrams for the beam using the method of superposition.



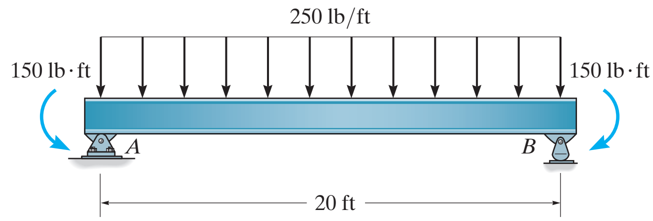


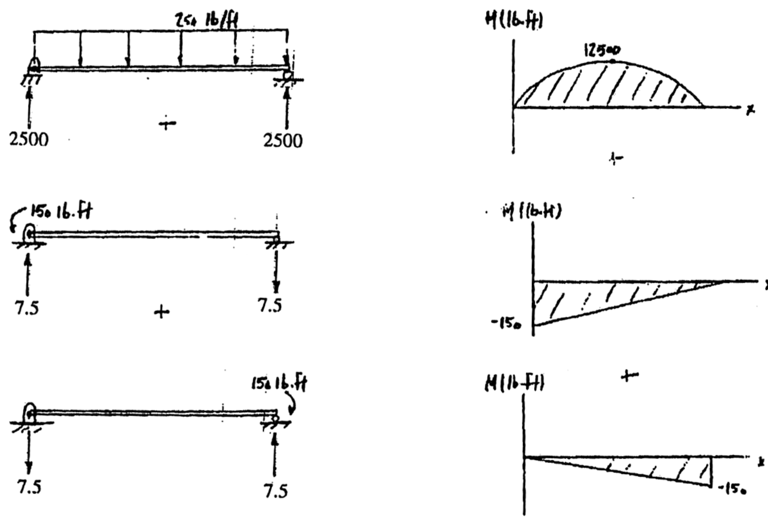
***4-52.** Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be cantilevered from end *A*.



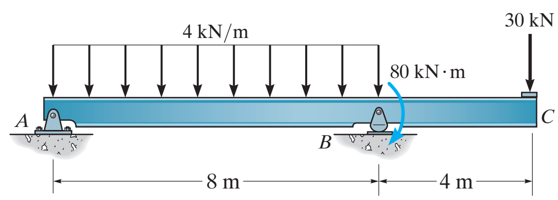


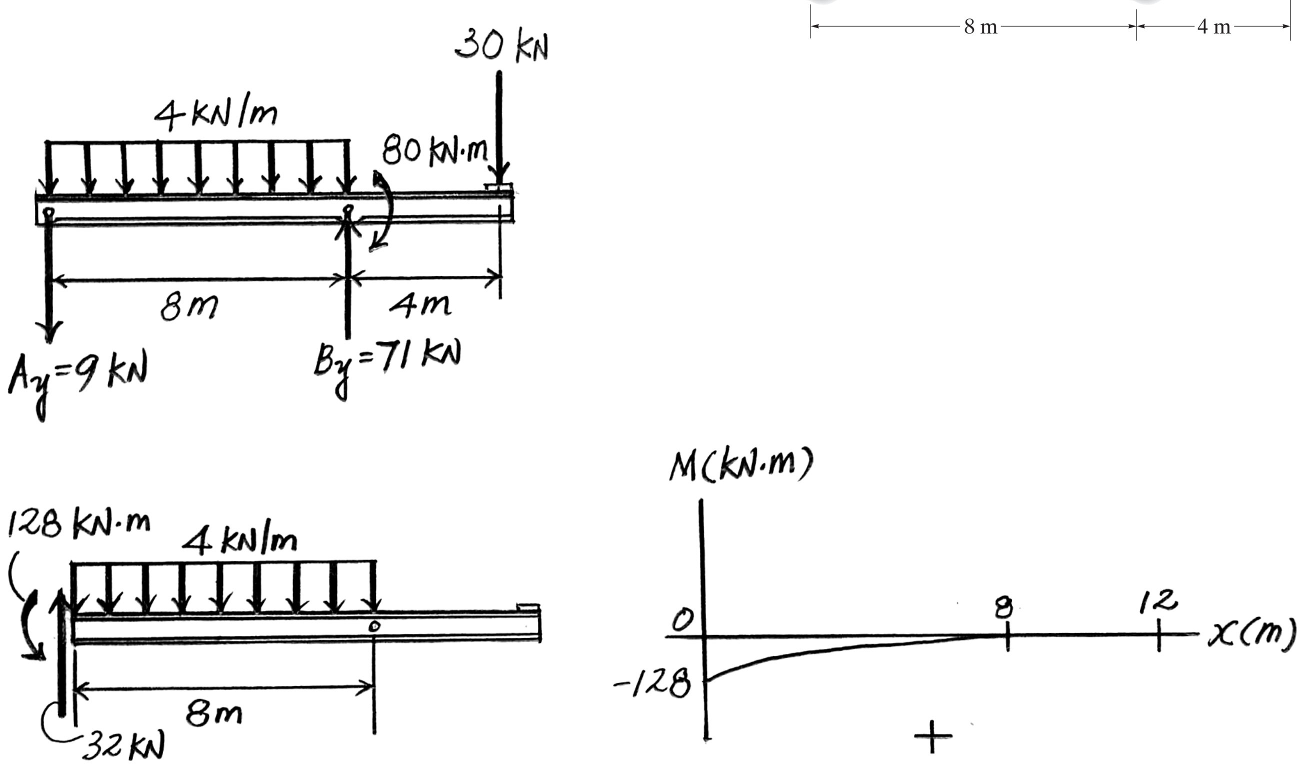
4–53. Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be simply supported at *A* and *B* as shown.



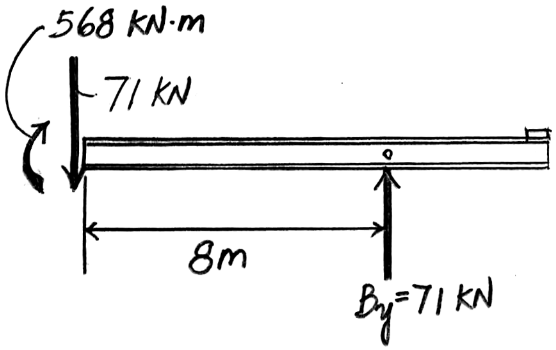


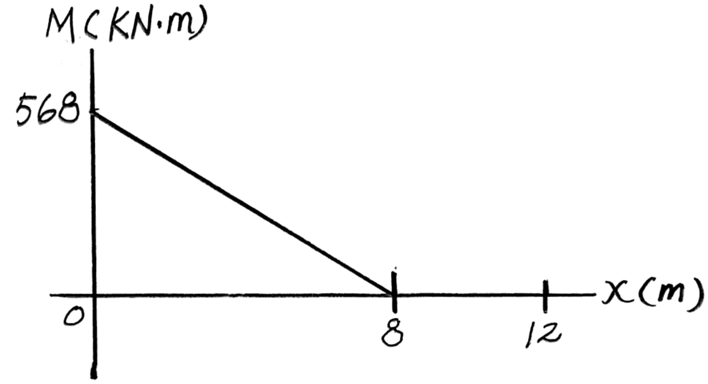
4–54. Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be cantilevered from the pin support at *A*.





4–54. Continued





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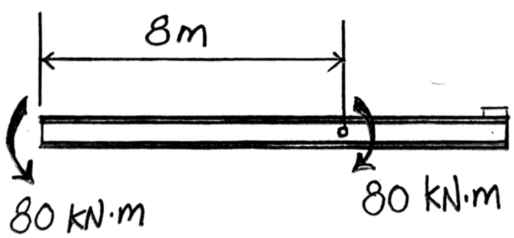

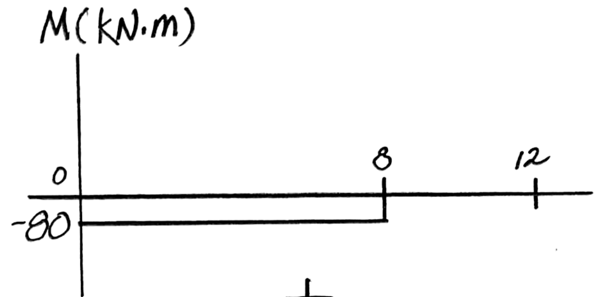

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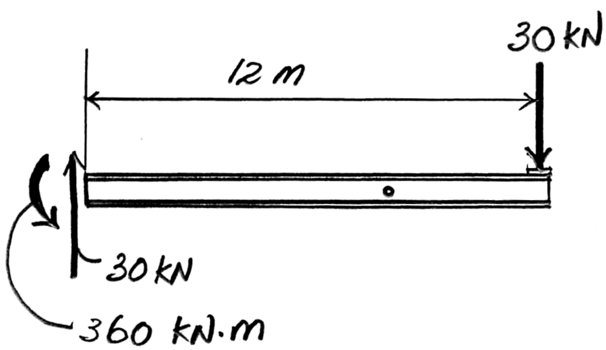
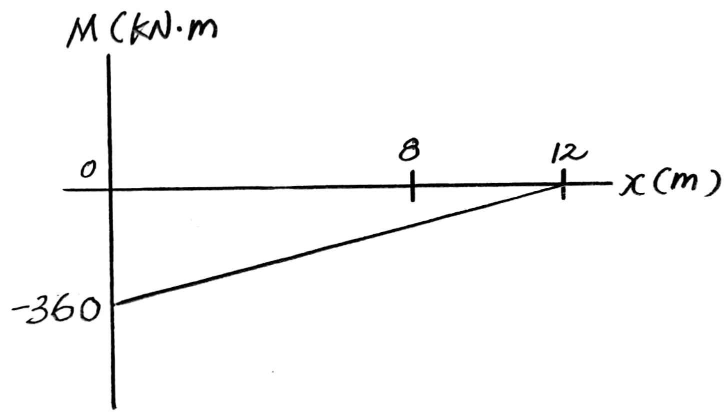
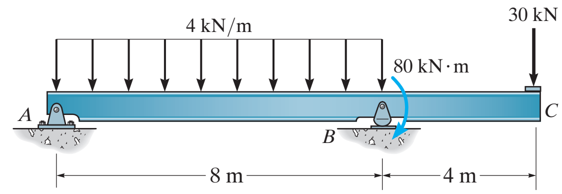
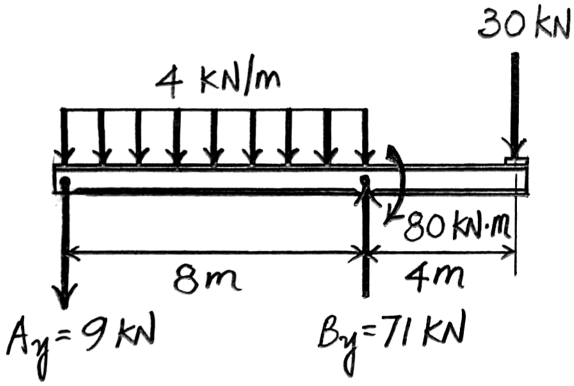
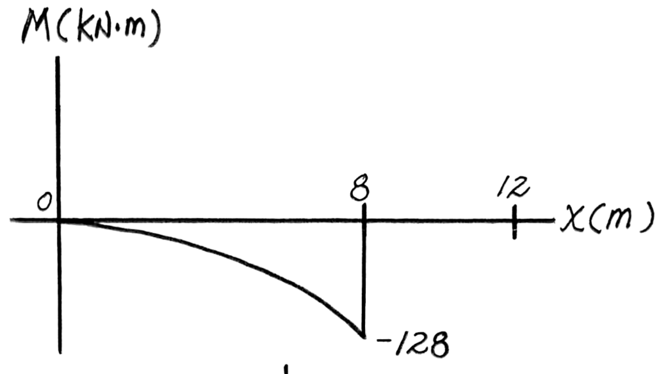




4–55. Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be cantilevered from the rocker at *B*.







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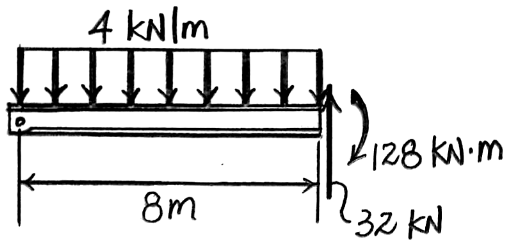

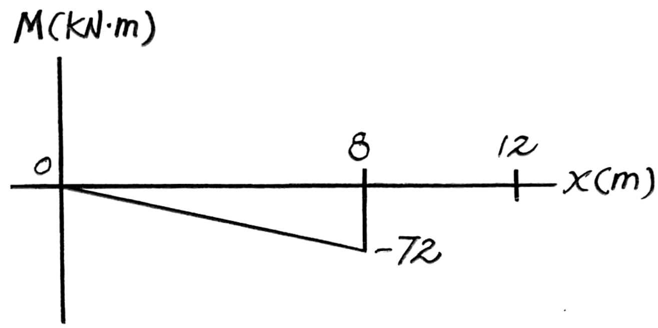

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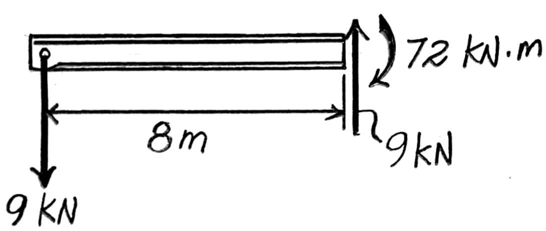
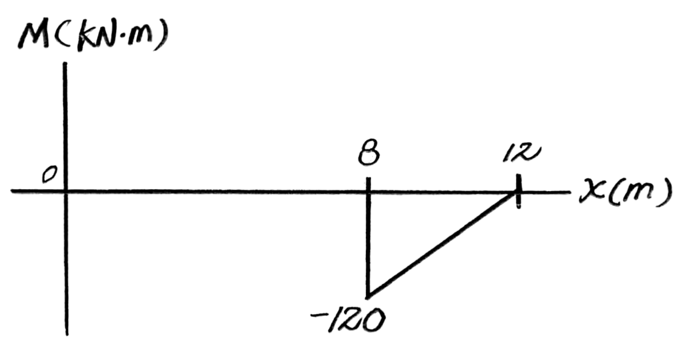
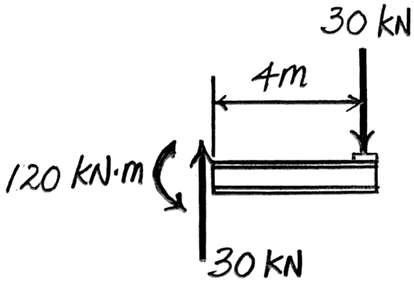
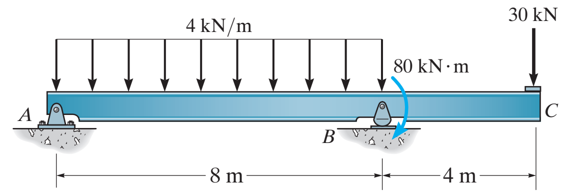
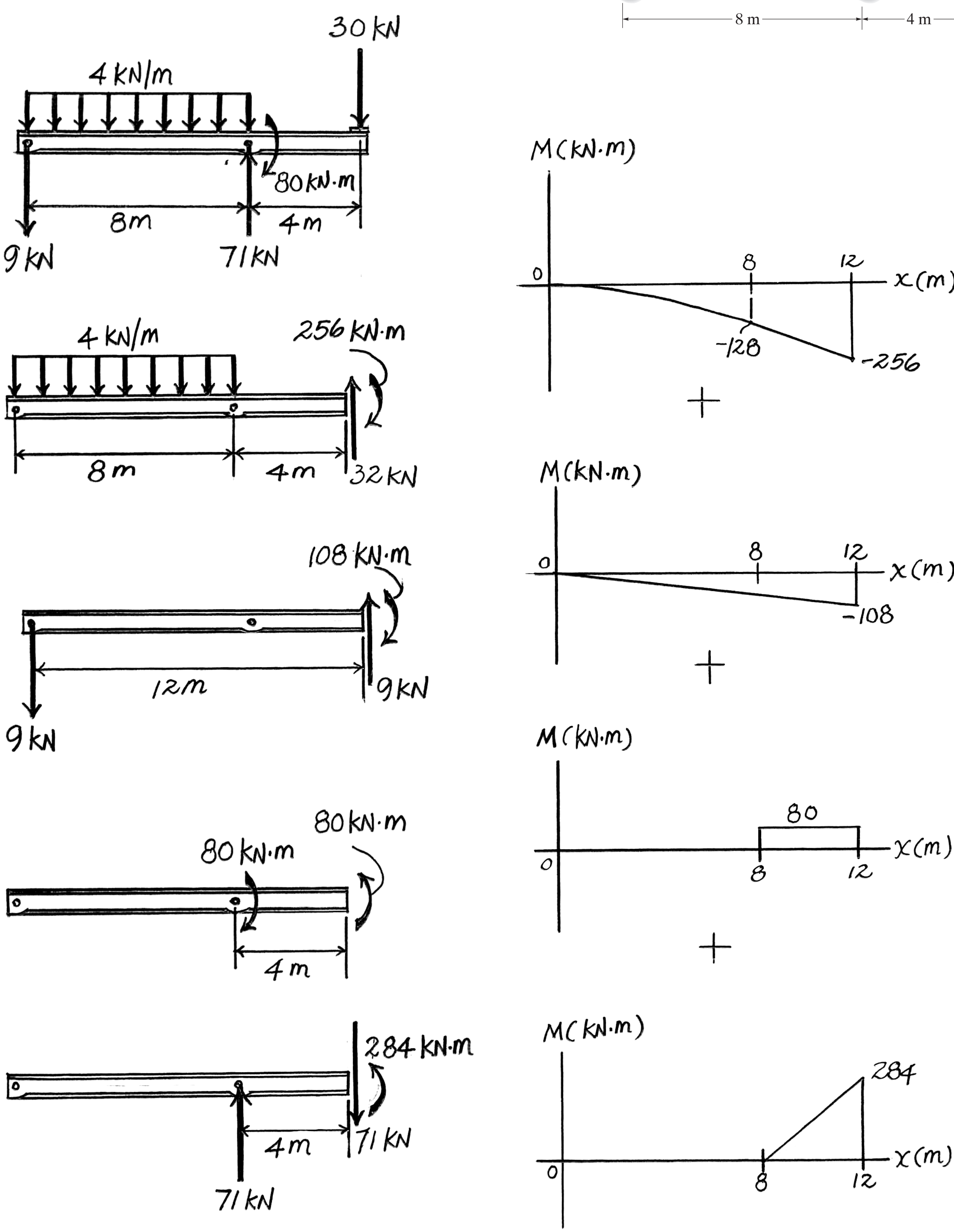






***4–56.** Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be cantilevered from end *C*.





4–57. Draw the moment diagrams for the beam using the method of superposition. Consider the beam to be simply supported at *A* and *B* as shown.